Homework problems:

1. Give regular expressions describing the following languages:
   (a) \( \{ w \in \{ a, b \}^* \mid w \text{ contains } abb \text{ as a substring} \} \);
   (b) \( \{ w \in \{ a, b \}^* \mid w \text{ contains either } abb \text{ or } bba \text{ (or both) as a substring} \} \);
   (c) \( \{ w \in \{ 0, 1 \}^* \mid w \text{ contains exactly two 0's} \} \);
   (d) \( \{ w \in \{ 0, 1 \}^* \mid w \text{ contains at least two 0's} \} \);
   (e) \( \{ w \in \{ 0, 1 \}^* \mid w \text{ contains an even number (possibly zero) of 0's} \} \);
   (f) \( \{ w \in \{ 0, 1 \}^* \mid |w| = 1 \pmod{3} \} \).

2. (a) Construct in a systematic way (as described in your textbook) a non-
deterministic finite automaton corresponding to the regular expression
   \( \varepsilon^* (001)^* (a^* b^*)^* \).
   (b) Make your automaton deterministic.
   (c) Describe the language in part (a) in English as simply as you can.

3. Give regular expressions describing the following languages:
   (a) \( \{ w \in \{ a, b \}^* \mid w \text{ does not contain } abb \text{ as a substring} \} \);
   (b) \( \{ w \in \{ 0, 1 \}^* \mid w \text{ begins and ends with different symbols} \} \);
   (Hint: Design first a finite automaton for each of the languages, cf. problems
   3/1(b) & 3/1(f), and convert these automata then in a systematic manner,
as described in your textbook, into the corresponding regular expressions.)

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions
describing the same languages):
   (a) \( \emptyset^* (a^* b^*)^* (b \cup a)b^* \)
   (b) \( (a \cup b)^* \emptyset (a \cup b)b^* a^* \)
   (c) \( a(b^* \cup a^*)(a^* b^*)^* \)

5. Determine whether the regular expressions \( r_1 = b^* a(a^* b^*)^* \) and \( r_2 = (a \cup b)^* a(a \cup b)^* \)
describe the same language, by constructing the minimal deterministic finite
automata corresponding to them.

6. Prove that if \( L \) is a regular language, then so is \( L' = \{ xy \mid x \in L, y \notin L \} \).