

**Introduction to Theoretical Computer Science**  
**Tutorial 5, 22–25 February**  
**Problems**

**Homework problems:**

1. Give regular expressions describing the following languages:
  - (a)  $\{w \in \{a, b\}^* \mid w \text{ contains } abb \text{ as a substring}\}$ ;
  - (b)  $\{w \in \{a, b\}^* \mid w \text{ contains either } abb \text{ or } bba \text{ (or both) as a substring}\}$ ;
  - (c)  $\{w \in \{0, 1\}^* \mid w \text{ contains exactly two 0's}\}$ ;
  - (d)  $\{w \in \{0, 1\}^* \mid w \text{ contains at least two 0's}\}$ ;
  - (e)  $\{w \in \{0, 1\}^* \mid w \text{ contains an even number (possibly zero) of 0's}\}$ ;
  - (f)  $\{w \in \{0, 1\}^* \mid |w| = 1 \pmod{3}\}$ .
2. (a) Construct in a systematic way (as described in your textbook) a non-deterministic finite automaton corresponding to the regular expression  $((\varepsilon \cup 0)1)^*011^*$ .
  - (b) Make your automaton deterministic.
  - (c) Describe the language in part (a) in English as simply as you can.
3. Give regular expressions describing the following languages:
  - (a)  $\{w \in \{a, b\}^* \mid w \text{ does not contain } abb \text{ as a substring}\}$ ;
  - (b)  $\{w \in \{0, 1\}^* \mid w \text{ begins and ends with different symbols}\}$ ;

(*Hint: Design first a finite automaton for each of the languages, cf. problems 3/1(b) & 3/1(f), and convert these automata then in a systematic manner, as described in your textbook, into the corresponding regular expressions.*)

**Demonstration problems:**

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
  - (a)  $(\emptyset^* \cup a)(a^*)(b \cup a)b^*$
  - (b)  $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
  - (c)  $a(b^* \cup a^*)(a^*b^*)^*$
5. Determine whether the regular expressions  $r_1 = b^*a(a^*b^*)^*$  and  $r_2 = (a \cup b)^*a(a \cup b)^*$  describe the same language, by constructing the minimal deterministic finite automata corresponding to them.
6. Prove that if  $L$  is a regular language, then so is  $L' = \{xy \mid x \in L, y \notin L\}$ .