Homework problems:

1. Give regular expressions describing the following languages:
   (a) \( \{ w \in \{ a, b \}^* | w \text{ starts with an } a \text{ and ends with a } b \} \)
   (b) \( \{ w \in \{ a, b \}^* | w \text{ contains either } ab \text{ or } ba \text{ (or both) as a substring} \} \)
   (c) \( \{ w \in \{ a, b \}^* | w \text{ contains the substring } ab \text{ an even number of times} \} \)
   (d) \( \{ w \in \{ a, \ldots, z, 0, \ldots, 9, \@ \}^* | w \text{ is a valid e-mail address} \} \)

2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression \((a a \cup b)^* bb\).
   (b) Make your automaton deterministic.

3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

   (a) 
   \[
   \begin{array}{c}
   (a) \quad b \quad a \\
   & b \\
   \end{array}
   \]

   (b) 
   \[
   \begin{array}{c}
   (b) \quad b \\
   & a \\
   \end{array}
   \]

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) \((\emptyset^* \cup a)(a^*)^*(b \cup a)b^*\)
   (b) \((a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*\)
   (c) \(a(b^* \cup a^*)(a^*b^*)^*\)

5. Determine whether the regular expressions \(r_1 = b^*a(a^*b^*)^*\) and \(r_2 = (a \cup b)^*a(a \cup b)^*\) describe the same language, by constructing the minimal deterministic finite automata corresponding to them.

6. Prove that if \(L\) is a regular language, then so is \(L' = \{ xy | x \in L, y \notin L \}\).