T-79.148 **Autumn 2004**

Introduction to Theoretical Computer Science

Tutorial 9, 16–17 November

Problems

Homework problems:

NB: The Turing machines requested in the following problems are most conveniently presented in diagram form.

1. Design a Turing machine that recognises ("decides") the language

$$\{w \in \{0,1\}^* \mid \text{the third-to-last symbol in } w \text{ is a } 1\}.$$

   Show the computation sequences ("runs") of your machine on inputs 0100, 100 and 00.

2. Design a Turing machine that replaces an input string $w \in \{a, b\}^*$ given on the tape by its lexicographic successor. I.e., if the input is *not* of the form $w = b^n$, then it is replaced by the alphabetically next string of length n; if on the other hand the input *is* of the form $w = b^n$, then it is replaced by a^{n+1}.

3. (a) Show that the language $\{wcw \mid w \in \{a, b\}^*\}$ is not context-free. (*Hint:* Consider strings of the form $a^n b^n c a^n b^n$.)

   (b) Design a Turing machine that recognises ("semidecides", or in this case even "decides") the above language.

Demonstration problems:

4. Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^k b^k c^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

5. Show that pushdown automata with *two* stacks (rather than just one as permitted by the standard definition) would be capable of recognising exactly the same languages as Turing machines.