Homework problems:

1. Convert the following grammar into Chomsky normal form:

   \[
   S \rightarrow AB \mid BA \mid \varepsilon \\
   A \rightarrow aS \\
   B \rightarrow bS
   \]

   Give also a simple verbal description of the language generated by the grammar.

2. Determine, using the CYK algorithm (“dynamic programming method”, Sipser p. 241, Lewis & Papadimitriou p. 155), whether the strings \(abab\), \(aabb\) and \(bbaab\) are generated by the grammar

   \[
   S \rightarrow AB \mid BA \mid a \mid b \\
   A \rightarrow BA \mid a \\
   B \rightarrow AB \mid b
   \]

   In the positive cases, give also the respective parse trees.

3. Design pushdown automata recognising the following languages:

   (a) \(\{wcw^R \mid w \in \{a,b\}^*\}\);
   (b) \(\{ww^R \mid w \in \{a,b\}^*\}\).

Demonstration problems:

4. Design an algorithm for testing whether a given a context-free grammar \(G = (V, \Sigma, P, S)\), generates a nonempty language, i.e. whether any terminal string \(x \in \Sigma^*\) can be derived from the start symbol \(S\).

5. Design a pushdown automaton corresponding to the grammar \(G = (V, \Sigma, P, S)\), where

   \[
   V = \{S, (, )^*, \cup, \emptyset, a, b\} \\
   \Sigma = \{(, ),^*, \cup, \emptyset, a, b\} \\
   P = \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\}
   \]

6. Design a grammar corresponding to the pushdown automaton \(M = (Q, \Sigma, \Gamma, \Delta, s, F)\), where

   \[
   Q = \{s, q, f\}, \Sigma = \{a, b\}, \Gamma = \{a, b, c\}, F = \{f\}, \\
   \Delta = \{(s, e, a), (q, c), (q, a, c), (q, ac), (q, a, a), (q, aa)\} \\
   \{(q, a, b), (q, e), (q, b, c), (q, bc), (q, b, b), (q, bb)\} \\
   \{(q, b, a), (q, e), (q, e, c), (f, e)\}\}