Homework problems:

1. Give regular expressions describing the following languages:
   (a) $\{ w \in \{a, b\}^* \mid w \text{ contains exactly two } a's \}$
   (b) $\{ w \in \{a, b\}^* \mid w \text{ contains at least two } a's \}$
   (c) $\{ w \in \{a, b\}^* \mid w \text{ contains an even number of } a's \}$
   (d) $\{ w \in \{a, b\}^* \mid w \text{ contains either } aa \text{ or } bb \text{ (or both) as a substring} \}$
   (e) $\{ w \in \{a, b\}^* \mid w \text{ contains neither } aa \text{ nor } bb \text{ as a substring} \}$
   (f) $\{ w \in \{0, 1\}^* \mid w \text{ begins and ends with different symbols} \}$
   (g) $\{ w \in \{0, 1\}^* \mid |w| = 1 \text{ (mod } 3) \}$
   (h) $\{ w \in \{a, \ldots, z, 0, \ldots, 9, @\}^* \mid w \text{ is a valid e-mail address} \}$
   (i) $\{ w \in \{a, \ldots, z, 0, \ldots, 9, @\}^* \mid w \text{ is a valid e-mail address ending in the country code 'fi' for Finland} \}$

2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression $((\varepsilon \cup 1)b)^*11^*$.
   (b) Make your automaton deterministic.
   (c) Describe the language in part (a) in English as simply as you can.

3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

   ![Automata Diagram](image)

   (a)

   (b)

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) $(\emptyset^* \cup a)(a^*)(b \cup a)b^*$
   (b) $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
   (c) $a(b^* \cup a^*)(a^+b^*)^*$

5. Determine whether the regular expressions $r_1 = b^*a(a^+b^*)^*$ and $r_2 = (a \cup b)^*a(a \cup b)^*$ describe the same language, by constructing the minimal deterministic finite automata corresponding to them.

6. Prove that if $L$ is a regular language, then so is $L' = \{ xy \mid x \in L, y \notin L \}$. 