Helsinki University of Technology
Laboratory for Theoretical Computer Science
Harri Haanpää (tel. 5243), Tommi Syrjänen (tel. 5082)

T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Sat 25 Oct 2003, 10 a.m. – 1 p.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 25.10.2003"
- The total number of answer sheets you are submitting for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:

   (a) $L = \{w \in \{0,1\}^* \mid w \text{ contains 010 as a substring}\}$; *7p.*

   (b) $\bar{L} = \{w \in \{0,1\}^* \mid w \text{ does not contain 010 as a substring}\}$. *8p.*

   Hint: It may be easiest to derive the solution to (b) from the solution to (a).

2. (a) Design a context-free grammar for the language

   $$S = \{a^m b^n c^{m+n} \mid m, n \geq 0\}.$$

   Draw the corresponding parse trees for the sentences *abcc* and *bbcc*. *8p.*

   (b) Prove (precisely!) that the language discussed in part (a) can not be described by a regular expression. *7p.*

3. Design a deterministic single-tape Turing machine that replaces an input string of the form $a^i b^j$, $i \geq j \geq 0$, given on its tape, by the string $A^i B^j C^k$, where $k = i - j$. Your machine does not need to check the validity of its input, i.e. you may assume that any given input string is of the indicated form. (Present the Turing machine preferably as a state diagram rather than a transition table.) Show the computation sequences ("runs") of your machine on inputs *aab*, *ab*, and *a*. *15p.*

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semi-decidable", "Turing-recognisable") language. What is the main difference between the two notions? *5p.*

   (b) Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.) *5p.*

   (c) Show that if a language $L \subseteq \Sigma^*$ is recursively enumerable but not recursive, then its complement language $\bar{L} = \Sigma^* - L$ is not recursively enumerable. (You may assume as known any auxiliary results related to this claim that have been presented in your textbook.) *5p.*

Total 60p.