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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Wed 7 May 2003, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 7.5.2003"
- The total number of answer sheets you are submitting for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:

   (a) $\{w \in \{0,1\}^* \mid w$ contains an even number (possibly zero) of 1's$\}$ *4p.*

   (b) $\{w \in \{0,1\}^* \mid w$ contains an odd number of 1's$\}$ *4p.*

   (c) $\{w \in \{0,1\}^* \mid w$ contains an even number (possibly zero) of 1's and ends in a 0, or contains an odd number of 1's and ends in a 1$\}$. *7p.*

   (*Hint:* In part (c) it may, depending on your solution method, be useful to first design a nondeterministic automaton.)

2. (a) Design a context-free grammar for the language

   $$L = \{a^n b^m \mid n \geq 0 \text{ and } m = n \text{ or } m = 2n\}.$$

   Draw the corresponding parse tree for the sentence *aabbbb*. *8p.*

   (b) Prove (precisely!) that the language L in part (a) is not regular. *7p.*

3. Design a (nondeterministic) pushdown automaton that recognises (accepts) the language L considered in problem 2. (Present the automaton preferably as a state diagram rather than a transition table.) Show the accepting computation sequences ("runs") of your automaton on the inputs *ab* and *abb*, and explain why it doesn't accept the input *aabbb*. *15p.*

4. *One* of the following:

   (a) Show that if the language L is context-free then so are the following languages:

   $$\begin{aligned} L^* &= \bigcup_{k \geq 0} L^k = \{w_1 \dots w_k \mid k \geq 0,\ w_i \in L \text{ for all } i = 1, \dots, k\}, \\ L^R &= \{w^R \mid w \in L\}. \end{aligned}$$

   (Here w^R denotes the *reversal* of string w, i.e. w written backwards.) *15p.*

   (b) Explain what is meant by a universal Turing machine, and outline at the level of basic operating principles the construction of such a machine. *15p.*

Total 60p.