Homework problems:
1. Give regular expressions describing the following languages:
   (a) \( \{ w \in \{a, b\}^* \mid w \text{ contains at least two } a \text{'s} \} \)
   (b) \( \{ w \in \{a, b\}^* \mid w \text{ contains either } aba \text{ or } bab \text{ (or both)} \text{ as a substring} \} \)
   (c) \( \{ w \in \{a, b\}^* \mid w \text{ begins and ends with different symbols} \} \)
   (d) \( \{ w \in \{1, 0\}^* \mid w \text{ contains neither } 11 \text{ nor } 00 \text{ as a substring} \} \)
2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression \( a(b \cup ab)^*b \).
   (b) Make your automaton deterministic.
3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:
   (a) \[
   \begin{array}{c}
   a &\xrightarrow{b} &b &\xrightarrow{a} &a &\xrightarrow{a} &a &\xrightarrow{b} &b \\
   b &\xrightarrow{a} &a &\xrightarrow{b} &b &\xrightarrow{b} &b &\xrightarrow{a} &a \\
   \end{array}
   \]
   (b) \[
   \begin{array}{c}
   b &\xrightarrow{a} &a &\xrightarrow{b} &b &\xrightarrow{a} &a &\xrightarrow{b} &b \\
   a &\xrightarrow{b} &b &\xrightarrow{a} &a &\xrightarrow{b} &b &\xrightarrow{a} &a \\
   \end{array}
   \]
Demonstration problems:
4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) \( (\emptyset^* \cup a)(a^*)^*(b \cup a)b^* \)
   (b) \( (a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^* \)
   (c) \( a(b^* \cup a^*)(a^*b^*)^* \)
5. Determine whether the regular expressions \( r_1 = b^*a(a^*b^*)^* \) and \( r_2 = (a \cup b)^*a(a \cup b)^* \) describe the same language, by constructing the minimal deterministic finite automata corresponding to them.
6. Prove that if \( L \) is a regular language, then so is \( L' = \{ xy \mid x \in L, y \notin L \} \).