T-79.148 **Spring 2002**
Introduction to Theoretical Computer Science
Tutorial 9, 26 March – 4 April
Problems

Homework problems:

NB: The Turing machines requested in the following problems are most conveniently presented in diagram form.

1. Design a Turing machine with input alphabet $\{a, b\}$ that accepts a given input string if and only if it contains at least two occurrences of the symbol a. Present the computation sequences of the machine on input strings aba and bab.

2. Design a Turing machine that increments a given binary number by one. More precisely, the machine is given as input a binary string x that it interprets as the binary representation of some integer n, and replaces with the binary representation of integer $n+1$. For simplicity, you may assume that binary numbers are represented on the machine's tape in "reverse order", i.e. with the least significant bits to the left and the most significant bits to the right.

3. (a) Show that the language $\{wcw \mid w \in \{a, b\}^*\}$ is not context-free. (*Hint:* Consider strings of the form $a^n b^n c a^n b^n$.)
   (b) Design a Turing machine that recognizes ("semidecides", or in this case even "decides") the above language.

Demonstration problems:

4. Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^k b^k c^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

5. Design Turing machines NEXT and DUP that perform the following tasks:
   (a) NEXT replaces a string given on the machine's tape by its immediate lexicographic successor;
   (b) DUP duplicates the string given on the tape, thus e.g. replacing the string abb by the string $abbabb$.