Homework problems:

1. Convert the grammar

\[ S \rightarrow (S) \mid A \]
\[ A \rightarrow SS \mid \varepsilon \]

into Chomsky normal form.

2. Determine, using the CYK algorithm ("dynamic programming method", Lewis & Papadimitriou p. 155), whether the strings \textit{aaaaa} and \textit{aaaaaa} are generated by the grammar

\[ S \rightarrow AB \mid BC \]
\[ A \rightarrow BA \mid a \]
\[ B \rightarrow CC \mid b \]
\[ C \rightarrow AB \mid a \]

In the positive case, give also the respective parse tree(s).

3. Design pushdown automata recognising the following languages:
   
   (a) \{ \textit{www}^R \mid w \in \{a, b\}^* \};
   
   (b) \{ \textit{ww}^R \mid w \in \{a, b\}^* \}.

Demonstration problems:

4. Given a context-free grammar \( G = (V, \Sigma, P, S) \), a nonterminal \( A \in V - \Sigma \) is redundant, if it cannot appear in the derivation of any sentence generated by \( G \), i.e. if no derivation in \( G \) is of the form \( S \Rightarrow^* \alpha A \beta \Rightarrow^* x \), where \( \alpha, \beta \in \Sigma^* \), \( x \in \Sigma^* \). Design an algorithm for removing all the redundant nonterminals from a grammar. (Hint: Determine first the \textit{non}redundant nonterminals.)

5. Design a pushdown automaton corresponding to the grammar \( G = (V, \Sigma, P, S) \), where

\[ V = \{S, (, ), *, \cup, \emptyset, a, b\} \]
\[ \Sigma = \{(, ), *, \cup, \emptyset, a, b\} \]
\[ P = \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\} \]

6. Design a grammar corresponding to the pushdown automaton \( M = (Q, \Sigma, \Gamma, \Delta, s, F) \), where

\[ Q = \{s, q, f\}, \Sigma = \{a, b\}, \Gamma = \{a, b, c\}, F = \{f\}, \]
\[ \Delta = \{(s, e, e), (q, c), ((q, a, c), (q, ac)), ((q, a, a), (q, aa)) \]
\[ ((q, a, b), (q, c)), ((q, b, c), (q, bc)), ((q, b, b), (q, bb)) \]
\[ ((q, b, a), (q, c)), ((q, e, c), (f, c)) \} \]