Homework problems:

1. Give regular expressions describing the following languages over the alphabet \{a, b\}:
   
   (a) \( \{w \in \{a, b\}^* \mid w \text{ contains exactly two } a's \} \)
   
   (b) \( \{w \in \{a, b\}^* \mid w \text{ contains at least two } a's \} \)
   
   (c) \( \{w \in \{a, b\}^* \mid w \text{ contains an even number of } a's \} \)
   
   (d) \( \{w \in \{a, b\}^* \mid w \text{ contains either } aa \text{ or } bb \text{ (or both) as a substring} \} \)
   
   (e) \( \{w \in \{a, b\}^* \mid w \text{ contains neither } aa \text{ nor } bb \text{ as a substring} \} \)
   
   (f) \( \{w \in \{a, b\}^* \mid w \text{ contains the substrings } ab \text{ and } ba \text{ (which may overlap)} \} \).

2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression \((ab \cup aab)^*\).
   
   (b) Make your automaton deterministic.

3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

   (a)
   
   ![Diagram A](image1)

   (b)
   
   ![Diagram B](image2)

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):

   (a) \((\emptyset^* \cup a)(a^*)^*(b \cup a)b^*\)
   
   (b) \((a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*\)
   
   (c) \(a(b^* \cup a^*)(a^*b^*)^*\)

5. Determine whether the regular expressions \(r_1 = b^*a(a^*b^*)^*\) and \(r_2 = (a \cup b)^*a(a \cup b)^*\) describe the same language, by constructing the (minimal) finite state machines corresponding to them.

6. Prove that if \(L\) is a regular language, then so is \(L' = \{xy \mid x \in L, y \notin L\}\).