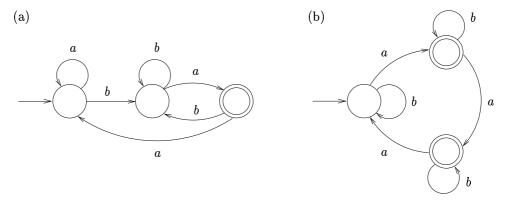
Introduction to Theoretical Computer Science Tutorial 5, 26-28 February Problems

Homework problems:

- 1. Give regular expressions describing the following languages over the alphabet $\{a, b\}$:
 - (a) $\{w \in \{a,b\}^* \mid w \text{ contains exactly two } a\text{'s}\}$
 - (b) $\{w \in \{a,b\}^* \mid w \text{ contains at least two } a\text{'s}\}$
 - (c) $\{w \in \{a, b\}^* \mid w \text{ contains an even number of } a$'s $\}$
 - (d) $\{w \in \{a,b\}^* \mid w \text{ contains either } aa \text{ or } bb \text{ (or both) as a substring}\}$
 - (e) $\{w \in \{a,b\}^* \mid w \text{ contains neither } aa \text{ nor } bb \text{ as a substring}\}$
 - (f) $\{w \in \{a,b\}^* \mid w \text{ contains the substrings } ab \text{ and } ba \text{ (which may overlap)}\}.$
- 2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression $(ab \cup aab)^*$.
 - (b) Make your automaton deterministic.
- 3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:



Demonstration problems:

- 4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
 - (a) $(\emptyset^* \cup a)(a^*)^*(b \cup a)b^*$
 - (b) $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
 - (c) $a(b^* \cup a^*)(a^*b^*)^*$
- 5. Determine whether the regular expressions $r_1 = b^* a(a^*b^*)^*$ and $r_2 = (a \cup b)^* a(a \cup b)^*$ describe the same language, by constructing the (minimal) finite state machines corresponding to them.
- 6. Prove that if L is a regular language, then so is $L' = \{xy \mid x \in L, y \notin L\}$.