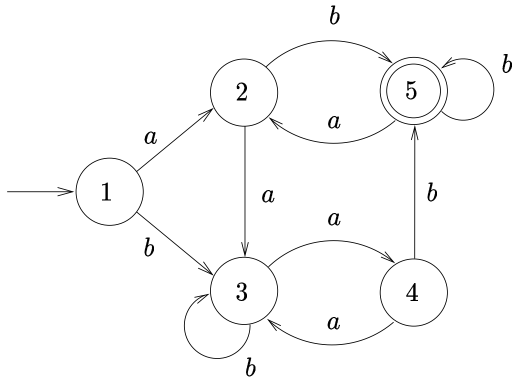
T-79.148 **Spring 2002**
Introduction to Theoretical Computer Science
Tutorial 4, 19-21 February
Problems

Homework problems:

1. Construct the minimal automaton corresponding to the following deterministic finite automaton:



2. Construct a nondeterministic finite automaton that tests whether a given input sequence over the alphabet $\{a, b\}$ contains $abaa$ as a subsequence. Make the automaton deterministic using the subset construction.

3. Show that if a language $L \subseteq \{a, b\}^*$ is recognized by some finite automaton, then so is the language $L^R = \{w^R \mid w \in L\}$. (The notation w^R means the reverse of string w, cf. problem 2/2.)

Demonstration problems:

4. Construct a nondeterministic finite automaton that tests whether in a given binary input sequence the third-to-last bit is a 1. Make the automaton deterministic using the subset construction.

5. Show that if languages A and B over the alphabet $\Sigma = \{a, b\}$ are recognized by some finite automata, then so are the languages $\bar{A} = \Sigma^* - A$, $A \cup B$, and $A \cap B$.

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6. *(Application.)* Many methods for analyzing data transfer protocols construct the state space of the system, which can be examined to find problems, e.g., deadlocks. One way of constructing the state space of the system is to model each participant of the protocol with a finite automaton and join these two into one big state machine.

Let $M_1 = (K_1, \Sigma_1, \Delta_1, s_1, \emptyset)$ and $M_2 = (K_2, \Sigma_2, \Delta_2, s_2, \emptyset)$ be nondeterministic automata. The joint state machine $M = (K, \Sigma, \Delta, s, \emptyset)$ is constructed in the following way:

- $K = K_1 \times K_2$
- $\Sigma = \Sigma_1 \cup \Sigma_2$
- $s = (s_1, s_2)$
- The transition $(p_1, p_2) \xrightarrow{a} (q_1, q_2)$ is in the relation Δ if any of the following conditions hold:
  (a) $a \in \Sigma_1 \cap \Sigma_2$, $(p_1, a, q_1) \in \Delta_1$ and $(p_2, a, q_2) \in \Delta_2$.
  (b) $a \in \Sigma_1$, $a \notin \Sigma_2$, $(p_1, a, q_1) \in \Delta_1$ and $p_2 = q_2$.
  (c) $a \notin \Sigma_1$, $a \in \Sigma_2$, $(p_2, a, q_2) \in \Delta_2$ and $p_1 = q_1$.

  Let M_1 and M_2 be as below. Construct the joint state machine M and show that the system has no deadlocks (i.e. from all states there is at least one transition)