Homework problems:

1. Design a finite automaton that models the behaviour of a simple TV set. The power switch of the TV has two alternative positions (on/off), the channel selector has three (1/2/3), and the voice control has two (hi/lo). The automaton does not need to have any distinct “final states”.

2. Design finite automata that recognise the following languages:
   
   (a) \( \{ w \in \{a,b\}^* \mid w \text{ contains } ab \text{ as a substring} \}; \)
   (b) \( \{ w \in \{a,b\}^* \mid w \text{ contains } aba \text{ as a substring} \}; \)
   (c) \( \{ w \in \{a,b\}^* \mid \text{ the last symbol of } w \text{ is } a \}; \)
   (d) \( \{ w \in \{a,b\}^* \mid \text{ the next to last symbol of } w \text{ is } a \}; \)
   (e) \( \{ w \in \{a,b\}^* \mid w \text{ contains an even number of } a \text{’s} \}. \)

3. Design a finite automaton that accepts precisely those binary strings that contain an even number of both 0’s and 1’s (e.g. 0011 and 1010, but not 001).

Demonstration problems:

4. Formulate the model of a simple coffee machine presented in class (lecture notes p. 15) precisely according to the mathematical definition of a finite automaton (Definition 2.1). What is the formal language recognized by this automaton?

5. Design finite automata that recognise the following languages:
   
   (a) \( \{ a^m b^n \mid m = n \mod 3 \}; \)
   (b) \( \{ w \in \{a,b\}^* \mid w \text{ contains equally many } a \text{’s and } b \text{’s, modulo 3} \}. \)

   (The notation “\( m = n \mod 3 \)” means that the numbers \( m \) and \( n \) yield the same remainder when divided by three.)

6. Design a finite automaton that recognizes sequences of integers separated by plus and minus signs (e.g. 11+20-9, -5+8). Implement your automaton as a computer program that also calculates the numerical value of the input expression.