T-79.148 **Spring 2002**
Introduction to Theoretical Computer Science
Tutorial 10, 9 – 11 April
Problems

Homework problems:

1. Design a two-tape Turing machine that recognizes the language $\{wcw \mid w \in \{a,b\}^*\}$. Construct the machine so that it makes use of both of its tapes. For simplicity, you may assume that one of the tape heads can stay stationary (direction code "S") while the other one moves.

2. Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on tape 3 the sum of the two given numbers in the same notation. For simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation $7 + 11 = 18$ is represented as:

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1110
1101
01001
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3. Design a nondeterministic Turing machine that recognizes the language $\{ww \mid w \in \{a,b\}^*\}$. How would you recognize the same language using a deterministic machine?

Demonstration problems:

4. Extend the notion of a Turing machine by providing the possibility of a *two-way infinite tape*. Show that nevertheless such machines recognize exactly the same languages as the standard machines whose tape is only one-way infinite.

5. Show that pushdown automata with *two* stacks (rather than just one as permitted by the standard definition) would be capable of recognizing exactly the same languages as Turing machines.