1. (a) Give a regular expression that describes the language
\{w \in \{0,1\}^* \mid w \text{ contains 0110 or 1001 as a substring (possibly both)}\}.

8p.

(b) Design a deterministic finite automaton that recognises the language in part (a).

7p.

2. (a) Design context-free grammars for the languages \(L \leq = \{a^i b^j \mid 0 \leq i \leq j\}\) and \(L \neq = \{a^i b^j \mid i \neq j\}\). (Hint: Note that \(i \neq j\) if and only if \(i < j\) or \(i > j\)).

6p.

(b) Prove (precisely!) that the language \(L \leq\) in part (a) is not regular.

6p.

(c) Prove (precisely!) that the language \(L \neq\) in part (a) is not regular.

3p.

3. Design a deterministic pushdown automaton that recognises (accepts) the language \(L \leq\) considered in problem 2. (Present the automaton preferably as a state diagram rather than a transition table.) Show the computation sequences of your automaton on the inputs \(abb\), \(bb\) and \(aba\).

15p.

4. One of the following:

(a) Show that if the language \(L \subseteq \{0,1\}^*\) is regular, then so are the following languages, consisting of all the prefixes and suffixes of the words in \(L\):

\[
\text{Pref}_L = \{x \in \{0,1\}^* \mid xy \in L \text{ for some } y \in \{0,1\}^*\},
\]

\[
\text{Suff}_L = \{y \in \{0,1\}^* \mid xy \in L \text{ for some } x \in \{0,1\}^*\}.
\]

15p.

(b) Formulate and state precisely so called “Rice’s theorem”, and apply it in some example case. (You do not need to prove the theorem, but you must define precisely all the concepts needed for its statement. Also in the application example you must indicate exactly how the theorem is being applied.)

15p.

Total 60p.