Homework problems:

1. Give regular expressions describing the following languages:
   (a) \( \{ w \in \{a, b\}^* \mid w \text{ contains exactly two } a's \} \)
   (b) \( \{ w \in \{a, b\}^* \mid w \text{ contains at least two } a's \} \)
   (c) \( \{ w \in \{a, b\}^* \mid w \text{ contains an even number of } a's \} \)
   (d) \( \{ w \in \{a, b\}^* \mid w \text{ contains either } aa \text{ or } bb \text{ (or both) as a substring} \} \)
   (e) \( \{ w \in \{a, b\}^* \mid w \text{ contains neither } aa \text{ nor } bb \text{ as a substring} \} \)
   (f) \( \{ w \in \{0, 1\}^* \mid \text{the third-to-last symbol in } w \text{ is 1} \} \)
   (g) \( \{ w \in \{a, \ldots, z, 0, \ldots, 9, @\}^* \mid w \text{ is a valid e-mail address} \} \)
   (h) \( \{ w \in \{a, \ldots, z, 0, \ldots, 9, @\}^* \mid w \text{ is a valid e-mail address ending in the country code } '.fi' \text{ for Finland} \} \)

2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression \( (a \cup b)^*a(a \cup b) \).
   (b) Make your automaton deterministic.

3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

   \[
   \begin{align*}
   (a) & \quad \begin{array}{c}
   \quad \vcenter{\hbox{\includegraphics[scale=0.5]{a_diagram}}}
   \end{array} \\
   (b) & \quad \begin{array}{c}
   \quad \vcenter{\hbox{\includegraphics[scale=0.5]{b_diagram}}}
   \end{array}
   \end{align*}
   \]

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) \( (\emptyset^* \cup a)(a^*)^*(b \cup a)b^* \)
   (b) \( (a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^* \)
   (c) \( a(b^* \cup a^*)(a^*b^*)^* \)

5. Determine whether the regular expressions \( r_1 = b^*a(a^*b^*)^* \) and \( r_2 = (a \cup b)^*a(a \cup b)^* \) describe the same language, by constructing the minimal deterministic finite automata corresponding to them.

6. Prove that if \( L \) is a regular language, then so is \( L' = \{ xy \mid x \in L, y \notin L \} \).