Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 8
Exercises

Ordinary exercises:

1. Let $M = (K, \Sigma, \delta, s, \{h\})$ be a Turing machine where

$$K = \{q_0, q_1, q_2, h\}$$
$$\Sigma = \{a, b, \sqcup, \triangleright\}$$
$$s = q_0$$

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	a	(q_0, b)	q_1	a	(q_2, a)	q_2	a	$(q_2, \rightarrow)$
q_0	b	$(q_0, \rightarrow)$	q_1	b	(q_1, b)	q_2	b	$(h, \sqcup)$
q_0	$\sqcup$	$(q_1, \rightarrow)$	q_1	$\sqcup$	$(h, \sqcup)$	q_2	$\sqcup$	$(h, \sqcup)$
q_0	$\triangleright$	$(q_0, \rightarrow)$	q_1	$\triangleright$	$(q_1, \rightarrow)$	q_2	$\triangleright$	$(q_2, \rightarrow)$

How does the machine operate when it starts from the initial configuration $(q_0, \triangleright\underline{a}b\sqcup aa)$? How about the configuration $(q_0, \triangleright\underline{a}b\sqcup ba)$?

2. Construct a Turing machine that reads the input string from left to right and stops when it has read the string abb and replaced it by the string bbb.

3. A Turing machine M *decides* a language L if:

$$(s, \triangleright\sqcup w\underline{\sqcup}) \vdash_M^* (h, \triangleright\sqcup Y\underline{\sqcup}) \text{ for all } w \in L, \text{ ja}$$
$$(s, \triangleright\sqcup w\underline{\sqcup}) \vdash_M^* (h, \triangleright\sqcup N\underline{\sqcup}) \text{ for all } w \notin L.$$

Construct a Turing machine that decides the language:

$$L(M) = \{w \in \{a\}^* \mid |w| = 1\} \ .$$

Demonstration exercises:

4. Let $M = (K, \Sigma, \delta, s, \{h\})$ be a Turing machine where:

$$K = \{q_0, q_1, q_2, h\}$$
$$\Sigma = \{a, \sqcup, \triangleright\}$$
$$s = q_0$$

q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$	q	σ	$\delta(q,\sigma)$
q_0	a	$(q_1, \leftarrow)$	q_1	a	$(q_2, \sqcup)$	q_2	a	(q_2, a)
q_0	$\sqcup$	$(q_0, \sqcup)$	q_1	$\sqcup$	$(h, \sqcup)$	q_2	$\sqcup$	$(q_0, \leftarrow)$
q_0	$\triangleright$	$(q_0, \rightarrow)$	q_1	$\triangleright$	$(q_1, \rightarrow)$	q_2	$\triangleright$	$(q_2, \rightarrow)$

How does M behave when it starts from the configuration $(q_0, \triangleright \sqcup a^n\underline{a})$, when $n \geq 0$?

5. Construct a Turing machine that decides the language:

$$L(M) = \{w \in \{a, b\}^* \mid \text{there is at least one } a \text{ in } w\ \} \ .$$