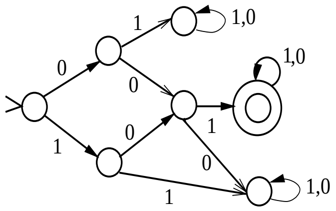
Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 4
Exercises

Ordinary Exercises:

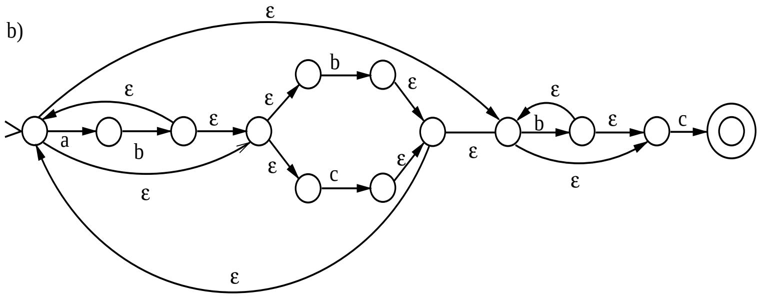
1. Draw the automata which accept the following languages.
   a) $(a^*(ba)^*b)^*a$
   b) $(q^* \cup f^*) \cup (f^*q^*aq^*(f \cup q))$
   c) $L = \{w \mid w \text{ is a binary number such that } w > 1101\}$.

2. Give regular expressions equivalent to the finite automata below. Specify whether the automaton is deterministic or not.

a)



b)



3. a) Construct a nondeterministic automaton equivalent to the regular expression $(a^* \cup b^*)ba^*(a \cup b)$.
   b) Make the automaton deterministic.

Demonstration exercises:

4. Draw finite automata equivalent to the following regular expressions:
   a) $(ab)^*(ba)^* \cup aa^*$
   b) $((ab \cup aab)^*a^*)^*$
   c) $((a^*b^*a^*)^*b)^*$
   d) $(ba \cup b)^* \cup (bb \cup a)^*$

5. A nondeterministic automaton can be defined in many ways. One way is $M = (K, \Sigma, \Delta, S, F)$, where K, Σ, Δ and F are as in the book, and S is a finite set of initial states, in the same way F is a finite state of final states. The automaton chooses its initial state nondeterministically. Explain why this new definition is more expressive than the one in the book.

6. Show that if M is a nondeterministic automaton, it follows that:

$$(q, xy) \vdash_M^* (p, y) \text{ if and only if } (q, x) \vdash_M^* (p, e)$$

7. (*application*)

   Many methods for analyzing data transfer protocols construct the state space of the system, which can be examined to find problems, e.g., deadlocks. One way of constructing the state space of the system is to model each participant of the protocol with a finite automaton and join these two into one big state machine.

   Let $M_1 = (K_1, \Sigma_1, \Delta_1, s_1, \emptyset)$ and $M_2 = (K_2, \Sigma_2, \Delta_2, s_2, \emptyset)$ be nondeterministic automata. The joint state machine $M = (K, \Sigma, \Delta, s, \emptyset)$ is constructed in the following way:

   - $K = K_1 \times K_2$
   - $\Sigma = \Sigma_1 \cup \Sigma_2$
   - $s = (s_1, s_2)$
   - The transition $(p_1, p_2) \xrightarrow{a} (q_1, q_2)$ is in the relation Δ if any of the following conditions hold:

     (a) $a \in \Sigma_1 \cap \Sigma_2$, $(p_1, a, q_1) \in \Delta_1$ and $(p_2, a, q_2) \in \Delta_2$.
     (b) $a \in \Sigma_1$, $a \notin \Sigma_2$, $(p_1, a, q_1) \in \Delta_1$ and $p_2 = q_2$.
     (c) $a \notin \Sigma_1$, $a \in \Sigma_2$, $(p_2, a, q_2) \in \Delta_2$ and $p_1 = q_1$.

     Let M_1 and M_2 be as below. Construct the joint state machine M and show that the system has no deadlocks (i.e. from all states there is at least one transition)