Ordinary exercises:

1. Simplify the following regular expressions:
   a) \((\emptyset^* \cup a)(a^*)^*(b \cup a)b^*\)
   b) \((a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*\)
   c) \(a(b^* \cup a^*)(a^*b^*)^*\)

2. Write the regular expressions over the alphabet \{0, 1\}, which describe the following languages:
   a) \(L = \{w \mid w \text{ has at most on pair of consequeutive ones}\}\)
   b) \(L = \{w \mid w \text{ has an even number of zeros}\}\)
   c) \(L = \{w \mid w \text{ does not contain the substring 101}\}\)

3. Which of the following statements are true? Why?
   a) \(aba \in (((c \cup b)^*a^*)^*(a^* \cup b^*)^*)^*\)
   b) \((a \cup b)^* = a^* \cup b^*\)
   c) \((a \cup b)^* \subseteq (a^*b^*)^*\)

Demonstration exercises:

4. Give a proof or a counterexample to the following statements.
   a) \(baa \in a^*b^*a^*b^*\)
   b) \(b^*a^* \cap a^*b^* = a^* \cup b^*\)
   c) \(a^*b^* \cap c^*d^* = \emptyset\)
   d) \(abcd \in (a(cd)^*b)^*\)

5. Show that \(a(b \cup c) = ab \cup ac\)

6. (Difficult) Show that if a language \(L\) is regular, then also the language \(L' = \{w \mid uw \in L \text{ for some string } u\}\) is regular.