Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 11
Exercises

Ordinary exercises: Let the function $odd : \mathbb{N} \to \mathbb{N}$ be defined as follows:

$$odd(n) = \begin{cases} 0 & , n \text{ is even} \\ 1 & , n \text{ is odd} \end{cases}$$

Show that $odd(n)$ is primitive recursive.

1. Let $f : \mathbb{N} \to \mathbb{N}$ be a primitive recursive function. Show that the function $F : \mathbb{N} \to \mathbb{N}$:
$$F(n) = f(f(f(\cdots f(n) \cdots))),$$
where f is composed with itself n times, is primitive recursive.

2. Show that the function:
$$prime(n) = \begin{cases} 1 & , n \text{ is prime} \\ 0 & , \text{otherwise} \end{cases}$$
is μ-recursive.

   The following functions that are known to be primitive recursive may help you in the proof:
$$iszero(n) = \begin{cases} 0 & , n > 0 \\ 1 & , n = 0 \end{cases} \qquad n \sim m = \begin{cases} n - m & , n > m \\ 0 & , n \leq m \end{cases}$$
   Additionally the addition, multiplication and comparison of two natural numbers are all primitive recursive.

Demonstration exercises:

3. Show that the function f is primitive recursive when $f(n)$ is the nth odd natural number.

4. Define the remainder of two natural numbers as a μ-recursive funtion. Use bounded minimization in your answer.

5. Let $\Delta = \{a, b, c\}, \beta = 4$ and
$$d_1 = a, \quad d_2 = b, \quad d_3 = c$$
   a) What is the Gödel number of the string abc in this system.
   b) What string corresponds to the Gödel number 19.

6. (*difficult*) The *kernel* of a directed graph $G = (V, E)$ is a set $K \subseteq V$ of nodes such that:
   (a) For all $v, u \in K$, the edge $(u, v) \notin E$ and
   (b) For all $v \notin K$ there exists $u \in K$ such that $(u, v) \in E$.

   Prove that the problem of finding a kernel is NP-complete.