Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 10
Exercises

Ordinary exercises:

1. Construct a two-tape Turing machine that decides the language:

$$\{a^n b^n c^n \mid n \geq 0\}.$$

Use both tapes during the computation.

Convention: the input is read and the answer is written on the first tape as in the case of a single-tape machine. The second tape is initially empty and its read/write head is positioned in the first tape position. It doesn't matter what the second tape contains in the end of the computation.

2. Form an unrestricted grammar (=type 0 grammar) G such that:

$$L(G) = \{ww \mid w \in \{a,b\}^*\}.$$

3. It can be proved that every unrestricted grammar G can be converted into an equivalent grammar G' such that all rules of G' are of the form: $uAv \to uwv$ where $A \in V - \Sigma$ and $u, v, w \in V^*$.

Do the conversion to the grammar $G = (V, \Sigma, R, S)$, where

$$\begin{aligned}
\Sigma &= \{a\}, \\
V &= \Sigma \cup \{S, [,], A, N\}, \text{ and} \\
R &= \{S \to [NA], S \to a, [N \to [NN, NA \to AAN, N] \to], [A \to a[, [] \to e\}.
\end{aligned}$$

Hint: Some of the rules are already in the desired form. You can convert the other rules by adding new non-terminals and splitting a rule into two or more new rules.

Demonstration exercises:

4. Construct an unrestricted grammar that generates the language:

$$L = \{a^{n^2} \mid n \geq 0\}$$

5. Prove that the following problem is undecidable:

Let M be a Turing machine. Does M stop when it is given the empty string e as input.

6. Show that the class of Turing acceptable languages is closed under union and intersection.