Ordinary exercises:

1. Construct a two-tape Turing machine that decides the language:

\[ \{a^n b^n c^n \mid n \geq 0\} \]

Use both tapes during the computation.

Convention: the input is read and the answer is written on the first tape as in the case of a single-tape machine. The second tape is initially empty and its read/write head is positioned in the first tape position. It doesn’t matter what the second tape contains in the end of the computation.

2. Form an unrestricted grammar (=type 0 grammar) \( G \) such that:

\[ L(G) = \{ww \mid w \in \{a, b\}^*\} \]

3. It can be proved that every unrestricted grammar \( G \) can be converted into an equivalent grammar \( G' \) such that all rules of \( G' \) are of the form:

\[ uAv \rightarrow uwv \] where \( A \in V - \Sigma \) and \( u, v, w \in V^* \).

Do the conversion to the grammar \( G = (V, \Sigma, R, S) \), where

\[
\begin{align*}
\Sigma &= \{a\}, \\
V &= \Sigma \cup \{S, [\cdot], A, N\}, \text{ and} \\
R &= \{S \rightarrow NA, S \rightarrow a, [N \rightarrow [NN, NA \rightarrow AAN, N] \rightarrow], [A \rightarrow a[\cdot] \rightarrow e].
\end{align*}
\]

Hint: Some of the rules are already in the desired form. You can convert the other rules by adding new non-terminals and splitting a rule into two or more new rules.

Demonstration exercises:

4. Construct an unrestricted grammar that generates the language:

\[ L = \{a^{n^2} \mid n \geq 0\} \]

5. Prove that the following problem is undecidable:

Let \( M \) be a Turing machine. Does \( M \) stop when it is given the empty string \( e \) as input.

6. Show that the class of Turing acceptable languages is closed under union and intersection.