Tik-79.148 **Spring 2001**
Introduction to Theoretical Computer Science
Tutorial 1
Exercises

Ordinary exercises:

1. Let $A = \{a, b, c\}$, $B = \{b, d\}$ ja $C = \{a, c, d, e\}$. Write the following sets explicitly:
   a) $A \cup (C - B)$
   b) $B \times (A \cap C)$
   c) $2^{\{\emptyset\}} - 2^{\emptyset}$

2. Let $A = \{a, b, c, d\}$ and a relation $R \subseteq A \times A$:
$$R = \{(a, a), (a, b), (a, d), (b, d), (b, c), (c, b), (d, d)\}$$
   Draw directed graphs representing the following relations:

a)	R	c)	$R \circ R$
b)	R^{-1}	d)	$R \cup (R \circ R)$

3. Let $A = \{a, b, c, d\}$ and $R \subseteq A \times A$:
$$R = \{(a, b), (b, c), (c, a), (c, c), (d, c)\}$$
   Draw directed graphs representing
   a) the transitive closure of R;
   b) the symmetric closure of R; and
   c) the reflexive transitive closure of R.

Demonstration exercises:

4. Let $f : A \to B$. Show that the following relation R is an equivalence relation on A: $(a, b) \in R$ if and only if $f(a) = f(b)$.

5. Prove that if S is any collection of sets, then $R_s = \{(A, B) \mid A, B \in S \text{ and } A \subseteq B\}$ is a partial order.

6. Is the transitive closure of a symmetric closure of a binary relation necessarily reflexive? Prove it or give a counterexample.

7. Let S be any set, and let $\mathcal{P}$ be the set of all partitions of S. Let R be the binary relation on $\mathcal{P}$ such that $(\Pi_1.\Pi_2) \in R$ if and only if for every $S_1 \in \Pi_1$, there is an $S_2 \in \Pi_2$ such that $S_1 \subseteq S_2$. Show that R is a partial order on $\mathcal{P}$. What elements of $\mathcal{P}$ are minimal and maximal?