

1. a)

1. $\langle 1 \rangle \neg(\Box P \rightarrow \Box(Q \rightarrow P))$
2. $\langle 1 \rangle \Box P$ (1)
3. $\langle 1 \rangle \neg\Box(Q \rightarrow P)$ (1)
4. $\langle 1, 2 \rangle \neg(Q \rightarrow P)$ (3)
5. $\langle 1, 2 \rangle Q$ (4)
6. $\langle 1, 2 \rangle \neg P$ (4)
7. $\langle 1, 2 \rangle P$ (2)

⊗

b)

1. $\langle 1 \rangle \neg(\Box(P \rightarrow Q) \rightarrow (\neg\Box\neg P \rightarrow \neg\Box\neg Q))$
2. $\langle 1 \rangle \Box(P \rightarrow Q)$ (1)
3. $\langle 1 \rangle \neg(\neg\Box\neg P \rightarrow \neg\Box\neg Q)$ (1)
4. $\langle 1 \rangle \neg\Box\neg P$ (3)
5. $\langle 1 \rangle \neg\neg\Box\neg Q$ (3)
6. $\langle 1 \rangle \Box\neg Q$ (5)
7. $\langle 1, 2 \rangle \neg\neg P$ (4)
8. $\langle 1, 2 \rangle P$ (7)
9. $\langle 1, 2 \rangle \neg Q$ (6)
10. $\langle 1, 2 \rangle P \rightarrow Q$ (2)
11. $\langle 1, 2 \rangle \neg P$ (10) | 12. $\langle 1, 2 \rangle Q$ (10)

⊗

⊗

c)

1. $\langle 1 \rangle \neg((\Box P \wedge \Box Q) \rightarrow \Box(P \wedge Q))$
2. $\langle 1 \rangle \Box P \wedge \Box Q$ (1)
3. $\langle 1 \rangle \neg\Box(P \wedge Q)$ (1)
4. $\langle 1 \rangle \Box P$ (2)
5. $\langle 1 \rangle \Box Q$ (2)
6. $\langle 1, 2 \rangle \neg(P \wedge Q)$ (3)
7. $\langle 1, 2 \rangle P$ (4)
8. $\langle 1, 2 \rangle Q$ (5)
9. $\langle 1, 2 \rangle \neg P$ (6) | 10. $\langle 1, 2 \rangle \neg Q$ (6)

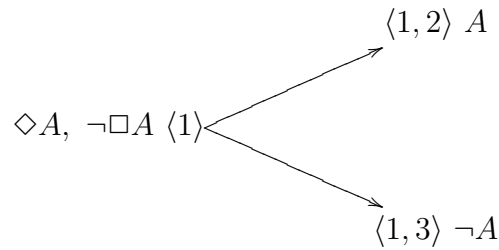
⊗

⊗

2. a)

1. $\langle 1 \rangle \neg(\Diamond A \rightarrow \Box A)$
2. $\langle 1 \rangle \Diamond A$ (1)
3. $\langle 1 \rangle \neg \Box A$ (1)
4. $\langle 1, 2 \rangle \neg \neg A$ (2; \Diamond on lyhennysmerkkintä $\neg \Box \neg$:lle)
5. $\langle 1, 2 \rangle A$ (4)
6. $\langle 1, 3 \rangle \neg A$ (3)

Ei **K**-pätevä.



b)

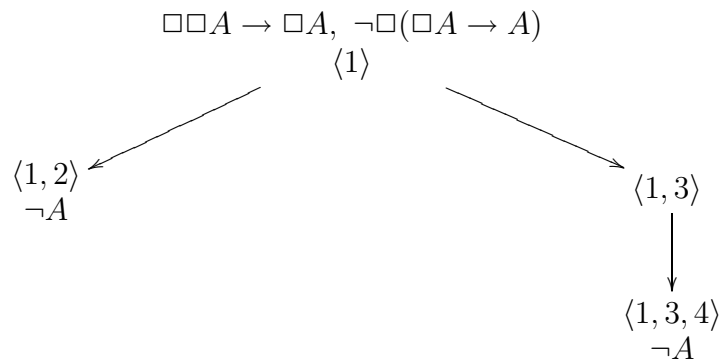
1. $\langle 1 \rangle \neg(\Diamond \Box A \vee \Box \Diamond \neg A)$
 2. $\langle 1 \rangle \neg \Diamond \Box A$ (1)
 3. $\langle 1 \rangle \neg \Box \Diamond \neg A$ (1)
 4. $\langle 1 \rangle \Box \neg \Box A$ (2)
 5. $\langle 1, 2 \rangle \neg \Diamond \neg A$ (3)
 6. $\langle 1, 2 \rangle \neg \Box A$ (4)
 7. $\langle 1, 2 \rangle \Box \neg \neg A$ (5)
 8. $\langle 1, 2, 3 \rangle \neg A$ (6)
 9. $\langle 1, 2, 3 \rangle \neg \neg A$ (7)
- ⊗

Lause on **K**-pätevä.

c)

1. $\langle 1 \rangle \neg((\Box \Box A \rightarrow \Box A) \rightarrow \Box(\Box A \rightarrow A))$
2. $\langle 1 \rangle \Box \Box A \rightarrow \Box A$ (1)
3. $\langle 1 \rangle \neg \Box(\Box A \rightarrow A)$ (1)
4. $\langle 1, 2 \rangle \neg(\Box A \rightarrow A)$ (3)
5. $\langle 1, 2 \rangle \Box A$ (4)
6. $\langle 1, 2 \rangle \neg A$ (4)
7. $\langle 1 \rangle \neg \Box \Box A$ (2) | 8. $\langle 1 \rangle \Box A$ (2)
9. $\langle 1, 3 \rangle \neg \Box A$ (7) | 11. $\langle 1, 2 \rangle A$ (8)
10. $\langle 1, 3, 4 \rangle \neg A$ (9) | ⊗

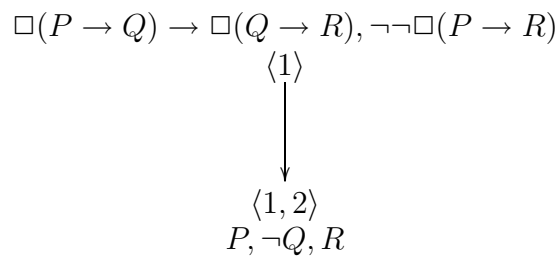
Ei **K**-pätevä.



3. a)

- | | | |
|-----|---|------|
| 1. | $\langle 1 \rangle \neg \left((\Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)) \rightarrow \neg\Box(P \rightarrow R) \right)$ | |
| 2. | $\langle 1 \rangle \Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)$ | (1) |
| 3. | $\langle 1 \rangle \neg\neg\Box(P \rightarrow R)$ | (1) |
| 4. | $\langle 1 \rangle \Box(P \rightarrow R)$ | (3) |
| 5. | $\langle 1 \rangle \neg\Box(P \rightarrow Q)$ | (2) |
| 6. | $\langle 1 \rangle \Box(Q \rightarrow R)$ | (2) |
| 7. | $\langle 1, 2 \rangle \neg(P \rightarrow Q)$ | (5) |
| 8. | $\langle 1, 2 \rangle P$ | (7) |
| 9. | $\langle 1, 2 \rangle \neg Q$ | (7) |
| 10. | $\langle 1, 2 \rangle P \rightarrow R$ | (4) |
| 11. | $\langle 1, 2 \rangle \neg P$ | (10) |
| 12. | $\langle 1, 2 \rangle R$ | (10) |
- ⊗

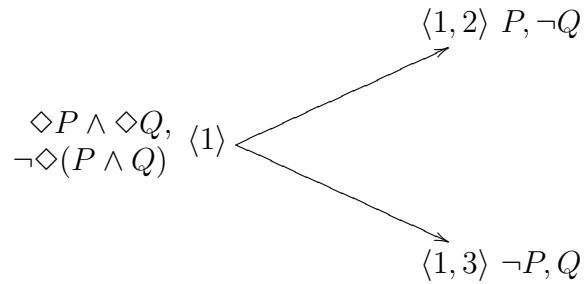
Ei **K**-pätevä.



b)

- | | | |
|-----------|--|---|
| 1. | $\langle 1 \rangle \neg ((\diamond P \wedge \diamond Q) \rightarrow \diamond(P \wedge Q))$ | |
| 2. | $\langle 1 \rangle \diamond P \wedge \diamond Q$ | (1) |
| 3. | $\langle 1 \rangle \neg \diamond(P \wedge Q)$ | (1) |
| 4. | $\langle 1 \rangle \diamond P$ | (2) |
| 5. | $\langle 1 \rangle \diamond Q$ | (2) |
| 6. | $\langle 1 \rangle \Box \neg(P \wedge Q)$ | (3) |
| 7. | $\langle 1, 2 \rangle P$ | (4) |
| 8. | $\langle 1, 2 \rangle \neg(P \wedge Q)$ | (6) |
| 9. | $\langle 1, 2 \rangle \neg P$ (8) | |
| \otimes | 10. | $\langle 1, 2 \rangle \neg Q$ (8) |
| | 11. | $\langle 1, 3 \rangle Q$ (5) |
| | 12. | $\langle 1, 3 \rangle \neg(P \wedge Q)$ (6) |
| | 13. | $\langle 1, 3 \rangle \neg P$ (11) 13. |
| | | $\langle 1, 3 \rangle \neg Q$ (12) |
| | | \otimes |

Ei **K**-pätevä.



c)

- | | | |
|-----------|--|---------------------------------------|
| 1. | $\langle 1 \rangle \neg (\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q))$ | |
| 2. | $\langle 1 \rangle \Box(P \wedge Q)$ | (1) |
| 3. | $\langle 1 \rangle \neg (\Box P \wedge \Box Q)$ | (1) |
| 4. | $\langle 1 \rangle \neg \Box P$ (3) | |
| 5. | $\langle 1 \rangle \neg \Box Q$ (3) | |
| 6. | $\langle 1, 2 \rangle \neg P$ (4) | |
| 7. | $\langle 1, 2 \rangle P \wedge Q$ (2) | |
| 8. | $\langle 1, 2 \rangle P$ (7) | |
| 9. | $\langle 1, 2 \rangle Q$ (7) | |
| \otimes | 10. | $\langle 1, 2 \rangle \neg Q$ (5) |
| | 11. | $\langle 1, 2 \rangle P \wedge Q$ (2) |
| | 12. | $\langle 1, 2 \rangle P$ (11) |
| | 13. | $\langle 1, 2 \rangle Q$ (11) |
| | | \otimes |