T-79.146 **Kevät 2004**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 6
Ratkaisut

Aksioomat ja päättelysäännöt Hilbert-tyylisiä todistuksia varten:

$$\begin{aligned}
\text{K:}&\quad \Box(P \to Q) \to (\Box P \to \Box Q)\\
\text{T:}&\quad \Box P \to P\\
\text{D:}&\quad \Box P \to \Diamond P \quad \text{tai} \quad \Box P \to \neg\Box\neg P\\
\text{4:}&\quad \Box P \to \Box\Box P\\
\text{5:}&\quad \neg\Box P \to \Box\neg\Box P\\
\text{MP:}&\quad \frac{P,\ P \to Q}{Q}\\
\text{N:}&\quad \frac{P}{\Box P}
\end{aligned}$$

1. a)

1.	$\neg\Box P \to \Box\neg\Box P$	[5-aksiooma]
2.	$(\neg\Box P \to \Box\neg\Box P) \to (\neg\Box\neg\Box P \to \Box P)$	[Tautologia]
3.	$\neg\Box\neg\Box P \to \Box P$	[MP, 1, 2]
4.	$\Box\Box P$	[GP]
5.	$\Box\Box P \to \neg\Box\neg\Box P$	[D]
6.	$\neg\Box\neg\Box P$	[MP, 4, 5]
7.	$\Box P$	[MP, 6, 3]

b) **S4** on **KT4**.

1.	$\neg\bot$	[Tautologia]
2.	$\Box\bot \to \bot$	[T]
3.	$(\Box\bot \to \bot) \to (\neg\bot \to \neg\Box\bot)$	[Tautologia]
4.	$\neg\bot \to \neg\Box\bot$	[MP, 2, 3]
5.	$\neg\Box\bot$	[MP, 1, 4]

2. a)

1.	$\langle 1\rangle\neg\big(\Box P \to \Box(Q \to P)\big)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box(Q \to P)$	(1)
4.	$\langle 1,2\rangle\neg(Q \to P)$	(3)
5.	$\langle 1,2\rangle Q$	(4)
6.	$\langle 1,2\rangle\neg P$	(4)
7.	$\langle 1,2\rangle P$	(2)
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Box(P \to Q) \to (\neg\Box\neg P \to \neg\Box\neg Q)\big)$	
2.	$\langle 1\rangle\Box(P \to Q)$	(1)
3.	$\langle 1\rangle\neg(\neg\Box\neg P \to \neg\Box\neg Q)$	(1)
4.	$\langle 1\rangle\neg\Box\neg P$	(3)
5.	$\langle 1\rangle\neg\neg\Box\neg Q$	(3)
6.	$\langle 1\rangle\Box\neg Q$	(5)
7.	$\langle 1,2\rangle\neg\neg P$	(4)
8.	$\langle 1,2\rangle P$	(7)
9.	$\langle 1,2\rangle\neg Q$	(6)
10.	$\langle 1,2\rangle P \to Q$	(2)
11.	$\langle 1,2\rangle\neg P$ (10) $\otimes$	12. $\langle 1,2\rangle Q$ (10) $\otimes$

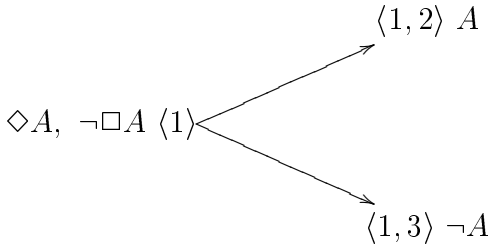
c)

1.	$\langle 1\rangle\neg\big((\Box P \wedge \Box Q) \to \Box(P \wedge Q)\big)$	
2.	$\langle 1\rangle\Box P \wedge \Box Q$	(1)
3.	$\langle 1\rangle\neg\Box(P \wedge Q)$	(1)
4.	$\langle 1\rangle\Box P$	(2)
5.	$\langle 1\rangle\Box Q$	(2)
6.	$\langle 1,2\rangle\neg(P \wedge Q)$	(3)
7.	$\langle 1,2\rangle P$	(4)
8.	$\langle 1,2\rangle Q$	(5)
9.	$\langle 1,2\rangle\neg P$ (6) $\otimes$	10. $\langle 1,2\rangle\neg Q$ (6) $\otimes$

3. a)

1.	$\langle 1\rangle\neg(\Diamond A \to \Box A)$	
2.	$\langle 1\rangle\Diamond A$	(1)
3.	$\langle 1\rangle\neg\Box A$	(1)
4.	$\langle 1,2\rangle\neg\neg A$	(2; $\Diamond$ on lyhennysmerkintä $\neg\Box\neg$:lle)
5.	$\langle 1,2\rangle A$	(4)
6.	$\langle 1,3\rangle\neg A$	(3)

Ei **K**-pätevä.



b)

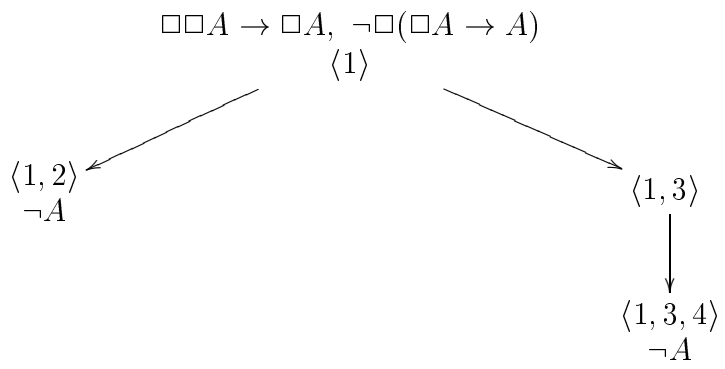
1.	$\langle 1\rangle\neg(\Diamond\Box A \vee \Box\Diamond\neg A)$	
2.	$\langle 1\rangle\neg\Diamond\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Diamond\neg A$	(1)
4.	$\langle 1\rangle\Box\neg\Box A$	(2)
5.	$\langle 1,2\rangle\neg\Diamond\neg A$	(3)
6.	$\langle 1,2\rangle\neg\Box A$	(4)
7.	$\langle 1,2\rangle\Box\neg\neg A$	(5)
8.	$\langle 1,2,3\rangle\neg A$	(6)
9.	$\langle 1,2,3\rangle\neg\neg A$	(7)
	$\otimes$	

Lause on **K**-pätevä.

c)

1.	$\langle 1\rangle\neg\big((\Box\Box A \to \Box A) \to \Box(\Box A \to A)\big)$				
2.	$\langle 1\rangle\Box\Box A \to \Box A$	(1)			
3.	$\langle 1\rangle\neg\Box(\Box A \to A)$	(1)			
4.	$\langle 1,2\rangle\neg(\Box A \to A)$	(3)			
5.	$\langle 1,2\rangle\Box A$	(4)			
6.	$\langle 1,2\rangle\neg A$	(4)			
7.	$\langle 1\rangle\neg\Box\Box A$	(2)	8.	$\langle 1\rangle\Box A$	(2)
9.	$\langle 1,3\rangle\neg\Box A$	(7)	11.	$\langle 1,2\rangle A$	(8)
10.	$\langle 1,3,4\rangle\neg A$	(9)		$\otimes$	

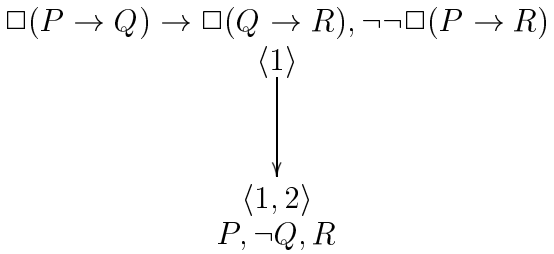
Ei **K**-pätevä.



4. a)

1.	$\langle 1\rangle\neg\Big(\big(\Box(P \to Q) \to \Box(Q \to R)\big) \to \neg\Box(P \to R)\Big)$					
2.	$\langle 1\rangle\Box(P \to Q) \to \Box(Q \to R)$			(1)		
3.	$\langle 1\rangle\neg\neg\Box(P \to R)$			(1)		
4.	$\langle 1\rangle\Box(P \to R)$			(3)		
5.	$\langle 1\rangle\neg\Box(P \to Q)$			(2)	6. $\langle 1\rangle\Box(Q \to R)$	(2)
7.	$\langle 1,2\rangle\neg(P \to Q)$			(5)		
8.	$\langle 1,2\rangle P$			(7)		
9.	$\langle 1,2\rangle\neg Q$			(7)		
10.	$\langle 1,2\rangle P \to R$			(4)		
11.	$\langle 1,2\rangle\neg P$	(10)	12.	$\langle 1,2\rangle R$	(10)	
	$\otimes$					

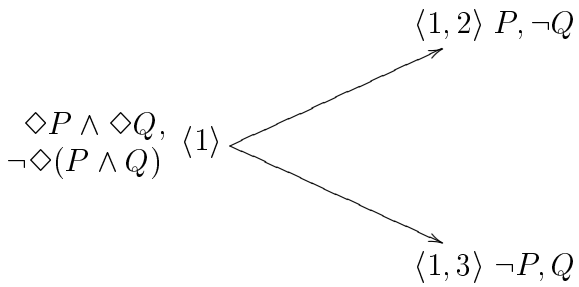
Ei **K**-pätevä.



b)

1.	$\langle 1\rangle\neg\big((\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(P \wedge Q)\big)$					
2.	$\langle 1\rangle\Diamond P \wedge \Diamond Q$				(1)	
3.	$\langle 1\rangle\neg\Diamond(P \wedge Q)$				(1)	
4.	$\langle 1\rangle\Diamond P$				(2)	
5.	$\langle 1\rangle\Diamond Q$				(2)	
6.	$\langle 1\rangle\Box\neg(P \wedge Q)$				(3)	
7.	$\langle 1,2\rangle P$				(4)	
8.	$\langle 1,2\rangle\neg(P \wedge Q)$				(6)	
9.	$\langle 1,2\rangle\neg P$	(8)	10.	$\langle 1,2\rangle\neg Q$	(8)	
	$\otimes$		11.	$\langle 1,3\rangle Q$	(5)	
			12.	$\langle 1,3\rangle\neg(P \wedge Q)$	(6)	
			13.	$\langle 1,3\rangle\neg P$	(11)	13. $\langle 1,3\rangle\neg Q$ (12)
						$\otimes$

Ei **K**-pätevä.



c)

1.	$\langle 1\rangle\neg\big(\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q)\big)$				
2.	$\langle 1\rangle\Box(P \wedge Q)$				(1)
3.	$\langle 1\rangle\neg(\Box P \wedge \Box Q)$				(1)
4.	$\langle 1\rangle\neg\Box P$	(3)	5.	$\langle 1\rangle\neg\Box Q$	(3)
6.	$\langle 1,2\rangle\neg P$	(4)	10.	$\langle 1,2\rangle\neg Q$	(5)
7.	$\langle 1,2\rangle P \wedge Q$	(2)	11.	$\langle 1,2\rangle P \wedge Q$	(2)
8.	$\langle 1,2\rangle P$	(7)	12.	$\langle 1,2\rangle P$	(11)
9.	$\langle 1,2\rangle Q$	(7)	13.	$\langle 1,2\rangle Q$	(11)
	$\otimes$			$\otimes$	