

Aksioomat ja päättelysäännöt Hilbert-tyylisiä todistuksia varten:

$$\text{K: } \Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$$

$$\text{T: } \Box P \rightarrow P$$

$$\text{D: } \Box P \rightarrow \Diamond P \quad \text{tai} \quad \Box P \rightarrow \neg \Box \neg P$$

$$4: \Box P \rightarrow \Box \Box P$$

$$5: \neg \Box P \rightarrow \Box \neg \Box P$$

$$\text{MP: } \frac{P, P \rightarrow Q}{Q}$$

$$\text{N: } \frac{P}{\Box P}$$

1. a)

- |    |   |              |
|----|---|--------------|
| 1. | $\neg \Box P \rightarrow \Box \neg \Box P$  | [5-aksioma]  |
| 2. | $(\neg \Box P \rightarrow \Box \neg \Box P) \rightarrow (\neg \Box \neg \Box P \rightarrow \Box P)$ | [Tautologia] |
| 3. | $\neg \Box \neg \Box P \rightarrow \Box P$  | [MP, 1, 2]   |
| 4. | $\Box \Box P$   | [GP]         |
| 5. | $\Box \Box P \rightarrow \neg \Box \neg \Box P$   | [D]          |
| 6. | $\neg \Box \neg \Box P$   | [MP, 4, 5]   |
| 7. | $\Box P$  | [MP, 6, 3]   |

b) S4 on KT4.

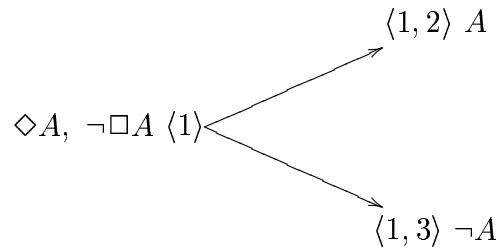
- |    |   |              |
|----|---|--------------|
| 1. | $\neg \perp$  | [Tautologia] |
| 2. | $\Box \perp \rightarrow \perp$  | [T]          |
| 3. | $(\Box \perp \rightarrow \perp) \rightarrow (\neg \perp \rightarrow \neg \Box \perp)$ | [Tautologia] |
| 4. | $\neg \perp \rightarrow \neg \Box \perp$  | [MP, 2, 3]   |
| 5. | $\neg \Box \perp$   | [MP, 1, 4]   |



3. a)

1.  $\langle 1 \rangle \neg(\Diamond A \rightarrow \Box A)$
2.  $\langle 1 \rangle \Diamond A$  (1)
3.  $\langle 1 \rangle \neg \Box A$  (1)
4.  $\langle 1, 2 \rangle \neg \neg A$  (2;  $\Diamond$  on lyhennysmerkinä  $\neg \Box \neg$ :lle)
5.  $\langle 1, 2 \rangle A$  (4)
6.  $\langle 1, 3 \rangle \neg A$  (3)

Ei **K**-pätevä.



b)

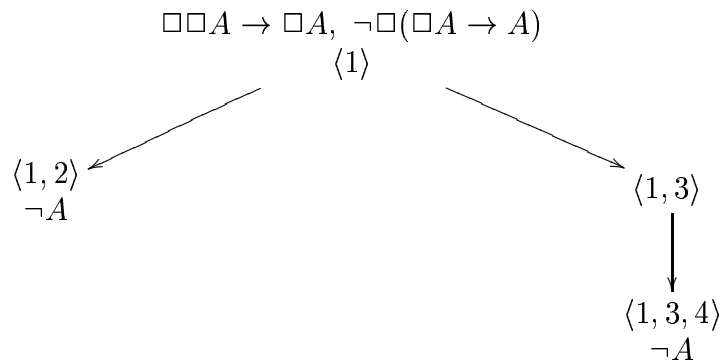
1.  $\langle 1 \rangle \neg(\Diamond \Box A \vee \Box \Diamond \neg A)$
  2.  $\langle 1 \rangle \neg \Diamond \Box A$  (1)
  3.  $\langle 1 \rangle \neg \Box \Diamond \neg A$  (1)
  4.  $\langle 1 \rangle \Box \neg \Box A$  (2)
  5.  $\langle 1, 2 \rangle \neg \Diamond \neg A$  (3)
  6.  $\langle 1, 2 \rangle \neg \Box A$  (4)
  7.  $\langle 1, 2 \rangle \Box \neg \neg A$  (5)
  8.  $\langle 1, 2, 3 \rangle \neg A$  (6)
  9.  $\langle 1, 2, 3 \rangle \neg \neg A$  (7)
- ⊗

Lause on **K**-pätevä.

c)

1.  $\langle 1 \rangle \neg((\Box \Box A \rightarrow \Box A) \rightarrow \Box(\Box A \rightarrow A))$
2.  $\langle 1 \rangle \Box \Box A \rightarrow \Box A$  (1)
3.  $\langle 1 \rangle \neg \Box(\Box A \rightarrow A)$  (1)
4.  $\langle 1, 2 \rangle \neg(\Box A \rightarrow A)$  (3)
5.  $\langle 1, 2 \rangle \Box A$  (4)
6.  $\langle 1, 2 \rangle \neg A$  (4)
7.  $\langle 1 \rangle \neg \Box \Box A$  (2) | 8.  $\langle 1 \rangle \Box A$  (2)
9.  $\langle 1, 3 \rangle \neg \Box A$  (7) | 11.  $\langle 1, 2 \rangle A$  (8)
10.  $\langle 1, 3, 4 \rangle \neg A$  (9) | ⊗

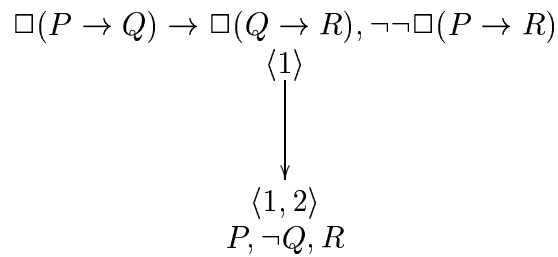
Ei **K**-pätevä.



4. a)

- |     |   |     |
|-----|---|-----|
| 1.  | $\langle 1 \rangle \neg \left( (\Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)) \rightarrow \neg\Box(P \rightarrow R) \right)$ |     |
| 2.  | $\langle 1 \rangle \Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)$   | (1) |
| 3.  | $\langle 1 \rangle \neg\neg\Box(P \rightarrow R)$   | (1) |
| 4.  | $\langle 1 \rangle \Box(P \rightarrow R)$   | (3) |
| 5.  | $\langle 1 \rangle \neg\Box(P \rightarrow Q)$   | (2) |
| 7.  | $\langle 1, 2 \rangle \neg(P \rightarrow Q)$  | (5) |
| 8.  | $\langle 1, 2 \rangle P$  | (7) |
| 9.  | $\langle 1, 2 \rangle \neg Q$   | (7) |
| 10. | $\langle 1, 2 \rangle P \rightarrow R$  | (4) |
| 11. | $\langle 1, 2 \rangle \neg P$ (10)  |     |
|     | 12. $\langle 1, 2 \rangle R$ (10)   | (2) |
- ⊗

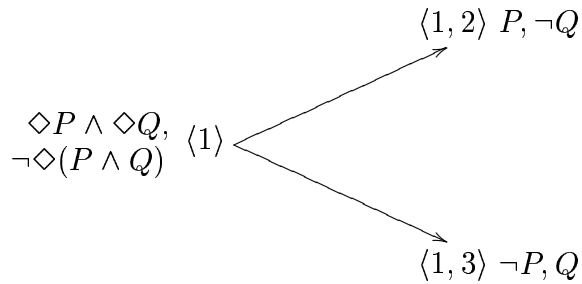
Ei **K**-pätevä.



b)

- |           |  |   |
|-----------|--|---|
| 1.        | $\langle 1 \rangle \neg ((\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(P \wedge Q))$ |   |
| 2.        | $\langle 1 \rangle \Diamond P \wedge \Diamond Q$   | (1)   |
| 3.        | $\langle 1 \rangle \neg \Diamond(P \wedge Q)$  | (1)   |
| 4.        | $\langle 1 \rangle \Diamond P$   | (2)   |
| 5.        | $\langle 1 \rangle \Diamond Q$   | (2)   |
| 6.        | $\langle 1 \rangle \Box \neg(P \wedge Q)$  | (3)   |
| 7.        | $\langle 1, 2 \rangle P$   | (4)   |
| 8.        | $\langle 1, 2 \rangle \neg(P \wedge Q)$  | (6)   |
| 9.        | $\langle 1, 2 \rangle \neg P$ (8)  |   |
| $\otimes$ | 10.  | $\langle 1, 2 \rangle \neg Q$ (8)           |
|           | 11.  | $\langle 1, 3 \rangle Q$ (5)                |
|           | 12.  | $\langle 1, 3 \rangle \neg(P \wedge Q)$ (6) |
|           | 13.  | $\langle 1, 3 \rangle \neg P$ (11)   13.    |
|           |  | $\langle 1, 3 \rangle \neg Q$ (12)          |
|           |  | $\otimes$                                   |

Ei **K**-pätevä.



c)

- |           |  |                                       |
|-----------|--|---------------------------------------|
| 1.        | $\langle 1 \rangle \neg (\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q))$ |                                       |
| 2.        | $\langle 1 \rangle \Box(P \wedge Q)$   | (1)                                   |
| 3.        | $\langle 1 \rangle \neg (\Box P \wedge \Box Q)$                                | (1)                                   |
| 4.        | $\langle 1 \rangle \neg \Box P$ (3)  | 5.                                    |
| 6.        | $\langle 1, 2 \rangle \neg P$ (4)  | 10.                                   |
| 7.        | $\langle 1, 2 \rangle P \wedge Q$ (2)  | 11.                                   |
| 8.        | $\langle 1, 2 \rangle P$ (7)   | 12.                                   |
| 9.        | $\langle 1, 2 \rangle Q$ (7)   | 13.                                   |
| $\otimes$ |  | $\langle 1 \rangle \neg \Box Q$ (3)   |
|           |  | $\langle 1, 2 \rangle \neg Q$ (5)     |
|           |  | $\langle 1, 2 \rangle P \wedge Q$ (2) |
|           |  | $\langle 1, 2 \rangle P$ (11)         |
|           |  | $\langle 1, 2 \rangle Q$ (11)         |
|           |  | $\otimes$                             |