**T-79.146** **Kevät 2003**
**Logiikka tietotekniikassa: erityiskysymyksiä I**
**Laskuharjoitus 6**
**Ratkaisut**

Aksioomat ja päättelysäännöt Hilbert-tyylisiä todistuksia varten:

K: $\Box(P \to Q) \to (\Box P \to \Box Q)$

T: $\Box P \to P$

D: $\Box P \to \Diamond P$ tai $\Box P \to \neg\Box\neg P$

4: $\Box P \to \Box\Box P$

5: $\neg\Box P \to \Box\neg\Box P$

MP: $\dfrac{P, P \to Q}{Q}$

N: $\dfrac{P}{\Box P}$

1. a)

| | | |
|---|---|---|
| 1. | $\neg\Box P \to \Box\neg\Box P$ | [5-aksiooma] |
| 2. | $(\neg\Box P \to \Box\neg\Box P) \to (\neg\Box\neg\Box P \to \Box P)$ | [Tautologia] |
| 3. | $\neg\Box\neg\Box P \to \Box P$ | [MP, 1, 2] |
| 4. | $\Box\Box P$ | [GP] |
| 5. | $\Box\Box P \to \neg\Box\neg\Box P$ | [D] |
| 6. | $\neg\Box\neg\Box P$ | [MP, 4, 5] |
| 7. | $\Box P$ | [MP, 6, 3] |

b) **S4** on **KT4**.

| | | |
|---|---|---|
| 1. | $\neg\bot$ | [Tautologia] |
| 2. | $\Box\bot \to \bot$ | [T] |
| 3. | $(\Box\bot \to \bot) \to (\neg\bot \to \neg\Box\bot)$ | [Tautologia] |
| 4. | $\neg\bot \to \neg\Box\bot$ | [MP, 2, 3] |
| 5. | $\neg\Box\bot$ | [MP, 1, 4] |

2. a)

| | | |
|---|---|---|
| 1. | $\langle 1\rangle\neg\big(\Box P \to \Box(Q \to P)\big)$ | |
| 2. | $\langle 1\rangle\Box P$ | (1) |
| 3. | $\langle 1\rangle\neg\Box(Q \to P)$ | (1) |
| 4. | $\langle 1,2\rangle\neg(Q \to P)$ | (3) |
| 5. | $\langle 1,2\rangle Q$ | (4) |
| 6. | $\langle 1,2\rangle\neg P$ | (4) |
| 7. | $\langle 1,2\rangle P$ | (2) |
| | $\otimes$ | |

b)

| | | |
|---|---|---|
| 1. | $\langle 1\rangle\neg\big(\Box(P \to Q) \to (\neg\Box\neg P \to \neg\Box\neg Q)\big)$ | |
| 2. | $\langle 1\rangle\Box(P \to Q)$ | (1) |
| 3. | $\langle 1\rangle\neg(\neg\Box\neg P \to \neg\Box\neg Q)$ | (1) |
| 4. | $\langle 1\rangle\neg\Box\neg P$ | (3) |
| 5. | $\langle 1\rangle\neg\neg\Box\neg Q$ | (3) |
| 6. | $\langle 1\rangle\Box\neg Q$ | (5) |
| 7. | $\langle 1,2\rangle\neg\neg P$ | (4) |
| 8. | $\langle 1,2\rangle P$ | (7) |
| 9. | $\langle 1,2\rangle\neg Q$ | (6) |
| 10. | $\langle 1,2\rangle P \to Q$ | (2) |
| 11. | $\langle 1,2\rangle\neg P$ (10) $\otimes$ | 12. $\langle 1,2\rangle Q$ (10) $\otimes$ |

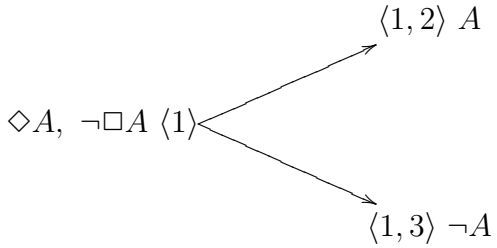
c)

| | | |
|---|---|---|
| 1. | $\langle 1\rangle\neg\big((\Box P \wedge \Box Q) \to \Box(P \wedge Q)\big)$ | |
| 2. | $\langle 1\rangle\Box P \wedge \Box Q$ | (1) |
| 3. | $\langle 1\rangle\neg\Box(P \wedge Q)$ | (1) |
| 4. | $\langle 1\rangle\Box P$ | (2) |
| 5. | $\langle 1\rangle\Box Q$ | (2) |
| 6. | $\langle 1,2\rangle\neg(P \wedge Q)$ | (3) |
| 7. | $\langle 1,2\rangle P$ | (4) |
| 8. | $\langle 1,2\rangle Q$ | (5) |
| 9. | $\langle 1,2\rangle\neg P$ (6) $\otimes$ | 10. $\langle 1,2\rangle\neg Q$ (6) $\otimes$ |

3. a)

| | | |
|---|---|---|
| 1. | $\langle 1\rangle\neg(\Diamond A \to \Box A)$ | |
| 2. | $\langle 1\rangle\Diamond A$ | (1) |
| 3. | $\langle 1\rangle\neg\Box A$ | (1) |
| 4. | $\langle 1,2\rangle\neg\neg A$ | (2; $\Diamond$ on lyhennysmerkintä $\neg\Box\neg$:lle) |
| 5. | $\langle 1,2\rangle A$ | (4) |
| 6. | $\langle 1,3\rangle\neg A$ | (3) |

Ei **K**-pätevä.




b)

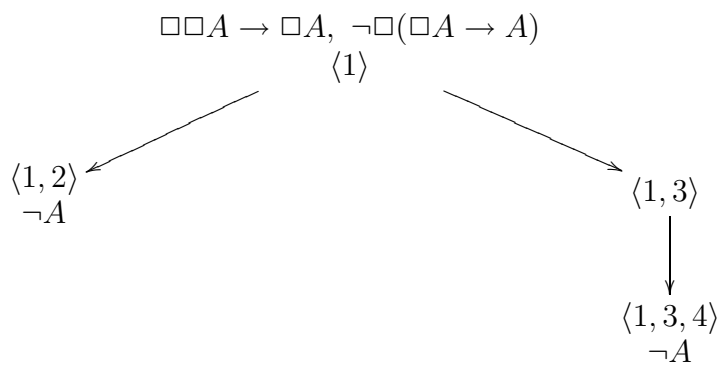
| | | |
|---|---|---|
| 1. | $\langle 1\rangle\neg(\Diamond\Box A \vee \Box\Diamond\neg A)$ | |
| 2. | $\langle 1\rangle\neg\Diamond\Box A$ | (1) |
| 3. | $\langle 1\rangle\neg\Box\Diamond\neg A$ | (1) |
| 4. | $\langle 1\rangle\Box\neg\Box A$ | (2) |
| 5. | $\langle 1,2\rangle\neg\Diamond\neg A$ | (3) |
| 6. | $\langle 1,2\rangle\neg\Box A$ | (4) |
| 7. | $\langle 1,2\rangle\Box\neg\neg A$ | (5) |
| 8. | $\langle 1,2,3\rangle\neg A$ | (6) |
| 9. | $\langle 1,2,3\rangle\neg\neg A$ | (7) |
| | $\otimes$ | |

Lause on **K**-pätevä.

c)

| | | | | | |
|---|---|---|---|---|---|
| 1. | $\langle 1\rangle\neg\big((\Box\Box A \to \Box A) \to \Box(\Box A \to A)\big)$ | | | | |
| 2. | $\langle 1\rangle\Box\Box A \to \Box A$ | (1) | | | |
| 3. | $\langle 1\rangle\neg\Box(\Box A \to A)$ | (1) | | | |
| 4. | $\langle 1,2\rangle\neg(\Box A \to A)$ | (3) | | | |
| 5. | $\langle 1,2\rangle\Box A$ | (4) | | | |
| 6. | $\langle 1,2\rangle\neg A$ | (4) | | | |
| 7. | $\langle 1\rangle\neg\Box\Box A$ | (2) | 8. | $\langle 1\rangle\Box A$ | (2) |
| 9. | $\langle 1,3\rangle\neg\Box A$ | (7) | 11. | $\langle 1,2\rangle A$ | (8) |
| 10. | $\langle 1,3,4\rangle\neg A$ | (9) | | $\otimes$ | |

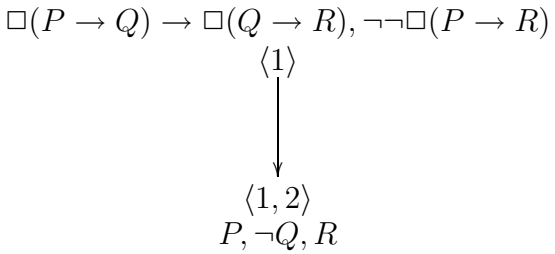
Ei **K**-pätevä.




4. a)

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | $\langle 1\rangle\neg\Big(\big(\Box(P \to Q) \to \Box(Q \to R)\big) \to \neg\Box(P \to R)\Big)$ | | | | | |
| 2. | $\langle 1\rangle\Box(P \to Q) \to \Box(Q \to R)$ | | (1) | | | |
| 3. | $\langle 1\rangle\neg\neg\Box(P \to R)$ | | (1) | | | |
| 4. | $\langle 1\rangle\Box(P \to R)$ | | (3) | | | |
| 5. | $\langle 1\rangle\neg\Box(P \to Q)$ | | (2) | 6. | $\langle 1\rangle\Box(Q \to R)$ | (2) |
| 7. | $\langle 1,2\rangle\neg(P \to Q)$ | | (5) | | | |
| 8. | $\langle 1,2\rangle P$ | | (7) | | | |
| 9. | $\langle 1,2\rangle\neg Q$ | | (7) | | | |
| 10. | $\langle 1,2\rangle P \to R$ | | (4) | | | |
| 11. | $\langle 1,2\rangle\neg P$ | (10) | | 12. | $\langle 1,2\rangle R$ | (10) |
| | $\otimes$ | | | | | |

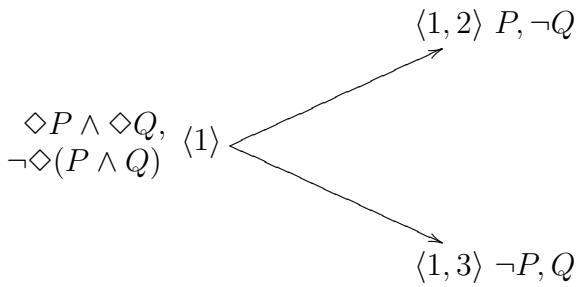
Ei **K**-pätevä.



b)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | $\langle 1\rangle\neg\big((\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(P \wedge Q)\big)$ | | | | | | |
| 2. | $\langle 1\rangle\Diamond P \wedge \Diamond Q$ | | | | (1) | | |
| 3. | $\langle 1\rangle\neg\Diamond(P \wedge Q)$ | | | | (1) | | |
| 4. | $\langle 1\rangle\Diamond P$ | | | | (2) | | |
| 5. | $\langle 1\rangle\Diamond Q$ | | | | (2) | | |
| 6. | $\langle 1\rangle\Box\neg(P \wedge Q)$ | | | | (3) | | |
| 7. | $\langle 1,2\rangle P$ | | | | (4) | | |
| 8. | $\langle 1,2\rangle\neg(P \wedge Q)$ | | | | (6) | | |
| 9. | $\langle 1,2\rangle\neg P$ | (8) | 10. | $\langle 1,2\rangle\neg Q$ | (8) | | |
| | $\otimes$ | | 11. | $\langle 1,3\rangle Q$ | (5) | | |
| | | | 12. | $\langle 1,3\rangle\neg(P \wedge Q)$ | (6) | | |
| | | | 13. | $\langle 1,3\rangle\neg P$ | (11) | 13. | $\langle 1,3\rangle\neg Q$ (12) |
| | | | | | | | $\otimes$ |

Ei **K**-pätevä.




c)

| | | | | | |
|---|---|---|---|---|---|
| 1. | $\langle 1\rangle\neg\big(\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q)\big)$ | | | | |
| 2. | $\langle 1\rangle\Box(P \wedge Q)$ | | | | (1) |
| 3. | $\langle 1\rangle\neg(\Box P \wedge \Box Q)$ | | | | (1) |
| 4. | $\langle 1\rangle\neg\Box P$ | (3) | 5. | $\langle 1\rangle\neg\Box Q$ | (3) |
| 6. | $\langle 1,2\rangle\neg P$ | (4) | 10. | $\langle 1,2\rangle\neg Q$ | (5) |
| 7. | $\langle 1,2\rangle P \wedge Q$ | (2) | 11. | $\langle 1,2\rangle P \wedge Q$ | (2) |
| 8. | $\langle 1,2\rangle P$ | (7) | 12. | $\langle 1,2\rangle P$ | (11) |
| 9. | $\langle 1,2\rangle Q$ | (7) | 13. | $\langle 1,2\rangle Q$ | (11) |
| | $\otimes$ | | | $\otimes$ | |