

Aksioomat ja päättelysäännöt Hilbert-tyylisiä todistuksia varten:

$$\text{K: } \Box(P \rightarrow Q) \rightarrow (\Box P \rightarrow \Box Q)$$

$$\text{T: } \Box P \rightarrow P$$

$$\text{D: } \Box P \rightarrow \Diamond P \quad \text{tai} \quad \Box P \rightarrow \neg \Box \neg P$$

$$4: \Box P \rightarrow \Box \Box P$$

$$5: \neg \Box P \rightarrow \Box \neg \Box P$$

$$\text{MP: } \frac{P, P \rightarrow Q}{Q}$$

$$\text{N: } \frac{P}{\Box P}$$

1. a)

- | | |
|--|--------------|
| 1. $\neg \Box P \rightarrow \Box \neg \Box P$ | [5-aksioma] |
| 2. $(\neg \Box P \rightarrow \Box \neg \Box P) \rightarrow (\neg \Box \neg \Box P \rightarrow \Box P)$ | [Tautologia] |
| 3. $\neg \Box \neg \Box P \rightarrow \Box P$ | [MP, 1, 2] |
| 4. $\Box \Box P$ | [GP] |
| 5. $\Box \Box P \rightarrow \neg \Box \neg \Box P$ | [D] |
| 6. $\neg \Box \neg \Box P$ | [MP, 4, 5] |
| 7. $\Box P$ | [MP, 6, 3] |

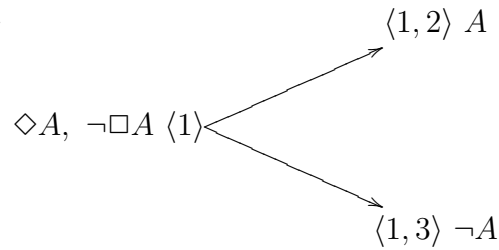
b) S4 on KT4.

- | | |
|--|--------------|
| 1. $\neg \perp$ | [Tautologia] |
| 2. $\Box \perp \rightarrow \perp$ | [T] |
| 3. $(\Box \perp \rightarrow \perp) \rightarrow (\neg \perp \rightarrow \neg \Box \perp)$ | [Tautologia] |
| 4. $\neg \perp \rightarrow \neg \Box \perp$ | [MP, 2, 3] |
| 5. $\neg \Box \perp$ | [MP, 1, 4] |

3. a)

1. $\langle 1 \rangle \neg(\Diamond A \rightarrow \Box A)$
2. $\langle 1 \rangle \Diamond A$ (1)
3. $\langle 1 \rangle \neg \Box A$ (1)
4. $\langle 1, 2 \rangle \neg \neg A$ (2; \Diamond on lyhennysmerkinä $\neg \Box \neg$:lle)
5. $\langle 1, 2 \rangle A$ (4)
6. $\langle 1, 3 \rangle \neg A$ (3)

Ei **K**-pätevä.



b)

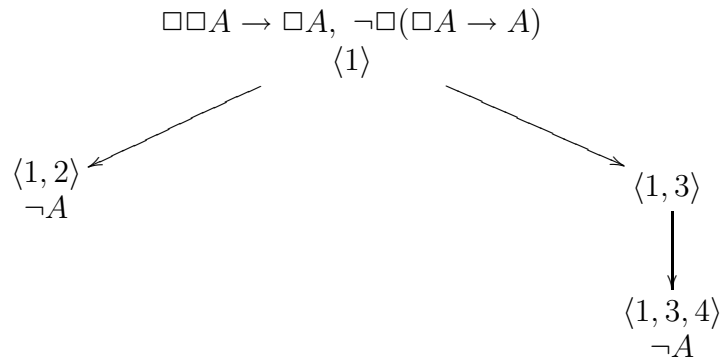
1. $\langle 1 \rangle \neg(\Diamond \Box A \vee \Box \Diamond \neg A)$
 2. $\langle 1 \rangle \neg \Diamond \Box A$ (1)
 3. $\langle 1 \rangle \neg \Box \Diamond \neg A$ (1)
 4. $\langle 1 \rangle \Box \neg \Box A$ (2)
 5. $\langle 1, 2 \rangle \neg \Diamond \neg A$ (3)
 6. $\langle 1, 2 \rangle \neg \Box A$ (4)
 7. $\langle 1, 2 \rangle \Box \neg \neg A$ (5)
 8. $\langle 1, 2, 3 \rangle \neg A$ (6)
 9. $\langle 1, 2, 3 \rangle \neg \neg A$ (7)
- ⊗

Lause on **K**-pätevä.

c)

1. $\langle 1 \rangle \neg((\Box \Box A \rightarrow \Box A) \rightarrow \Box(\Box A \rightarrow A))$
2. $\langle 1 \rangle \Box \Box A \rightarrow \Box A$ (1)
3. $\langle 1 \rangle \neg \Box(\Box A \rightarrow A)$ (1)
4. $\langle 1, 2 \rangle \neg(\Box A \rightarrow A)$ (3)
5. $\langle 1, 2 \rangle \Box A$ (4)
6. $\langle 1, 2 \rangle \neg A$ (4)
7. $\langle 1 \rangle \neg \Box \Box A$ (2) | 8. $\langle 1 \rangle \Box A$ (2)
9. $\langle 1, 3 \rangle \neg \Box A$ (7) | 11. $\langle 1, 2 \rangle A$ (8)
10. $\langle 1, 3, 4 \rangle \neg A$ (9) | ⊗

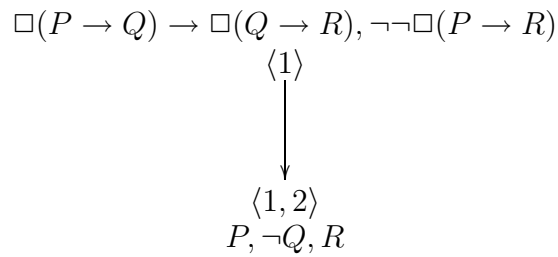
Ei **K**-pätevä.



4. a)

1. $\langle 1 \rangle \neg \left((\Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)) \rightarrow \neg\Box(P \rightarrow R) \right)$
 2. $\langle 1 \rangle \Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)$ (1)
 3. $\langle 1 \rangle \neg\neg\Box(P \rightarrow R)$ (1)
 4. $\langle 1 \rangle \Box(P \rightarrow R)$ (3)
 5. $\langle 1 \rangle \neg\Box(P \rightarrow Q)$ (2)
 6. $\langle 1 \rangle \Box(Q \rightarrow R)$ (2)
 7. $\langle 1, 2 \rangle \neg(P \rightarrow Q)$ (5)
 8. $\langle 1, 2 \rangle P$ (7)
 9. $\langle 1, 2 \rangle \neg Q$ (7)
 10. $\langle 1, 2 \rangle P \rightarrow R$ (4)
 11. $\langle 1, 2 \rangle \neg P$ (10) | 12. $\langle 1, 2 \rangle R$ (10)
- ⊗

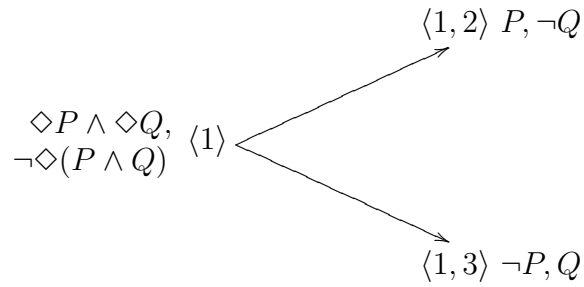
Ei **K**-pätevä.



b)

- | | | |
|-----------|---|---|
| 1. | $\langle 1 \rangle \neg((\diamond P \wedge \diamond Q) \rightarrow \diamond(P \wedge Q))$ | |
| 2. | $\langle 1 \rangle \diamond P \wedge \diamond Q$ | (1) |
| 3. | $\langle 1 \rangle \neg \diamond(P \wedge Q)$ | (1) |
| 4. | $\langle 1 \rangle \diamond P$ | (2) |
| 5. | $\langle 1 \rangle \diamond Q$ | (2) |
| 6. | $\langle 1 \rangle \Box \neg(P \wedge Q)$ | (3) |
| 7. | $\langle 1, 2 \rangle P$ | (4) |
| 8. | $\langle 1, 2 \rangle \neg(P \wedge Q)$ | (6) |
| 9. | $\langle 1, 2 \rangle \neg P$ (8) | |
| \otimes | 10. | $\langle 1, 2 \rangle \neg Q$ (8) |
| | 11. | $\langle 1, 3 \rangle Q$ (5) |
| | 12. | $\langle 1, 3 \rangle \neg(P \wedge Q)$ (6) |
| | 13. | $\langle 1, 3 \rangle \neg P$ (11) 13. |
| | | $\langle 1, 3 \rangle \neg Q$ (12) |
| | | \otimes |

Ei **K**-pätevä.



c)

- | | | |
|-----------|---|---------------------------------------|
| 1. | $\langle 1 \rangle \neg(\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q))$ | |
| 2. | $\langle 1 \rangle \Box(P \wedge Q)$ | (1) |
| 3. | $\langle 1 \rangle \neg(\Box P \wedge \Box Q)$ | (1) |
| 4. | $\langle 1 \rangle \neg \Box P$ (3) | |
| 5. | $\langle 1 \rangle \neg \Box Q$ (3) | |
| 6. | $\langle 1, 2 \rangle \neg P$ (4) | |
| 7. | $\langle 1, 2 \rangle P \wedge Q$ (2) | |
| 8. | $\langle 1, 2 \rangle P$ (7) | |
| 9. | $\langle 1, 2 \rangle Q$ (7) | |
| \otimes | 10. | $\langle 1, 2 \rangle \neg Q$ (5) |
| | 11. | $\langle 1, 2 \rangle P \wedge Q$ (2) |
| | 12. | $\langle 1, 2 \rangle P$ (11) |
| | 13. | $\langle 1, 2 \rangle Q$ (11) |
| | | \otimes |