T-79.146 **Kevät 2002**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 7
Ratkaisut

1. a)

1.	$\langle 1\rangle\neg(\Box P \to \Box\Box\Box\Box P)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box\Box\Box\Box P$	(1)
4.	$\langle 1,2\rangle\neg\Box\Box\Box P$	(3)
5.	$\langle 1,2,3\rangle\neg\Box\Box P$	(4)
6.	$\langle 1,2,3,4\rangle\neg\Box P$	(5)
7.	$\langle 1,2,3,4,5\rangle\neg P$	(6)
8.	$\langle 1,2,3,4,5\rangle P$	(2) (transitiivisuus)
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Diamond\neg P \to (\Diamond P \to \Diamond\Diamond P)\big)$	
2.	$\langle 1\rangle\Diamond\neg P$	(1)
3.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$	(1)
4.	$\langle 1\rangle\Diamond P$	(3)
5.	$\langle 1\rangle\neg\Diamond\Diamond P$	(3)
6.	$\langle 1,2\rangle P$	(4)
7.	$\langle 1,2\rangle\neg\Diamond P$	(5)
8.	$\langle 1,2\rangle\neg P$	(7) (refleksiivisyys)
	$\otimes$	

c)

1.	$\langle 1\rangle\neg\big(\Box\Box(P \to \Box Q) \to \Box(\Diamond P \to Q)\big)$				
2.	$\langle 1\rangle\Box\Box(P \to \Box Q)$				(1)
3.	$\langle 1\rangle\neg\Box(\Diamond P \to Q)$				(1)
4.	$\langle 1,2\rangle\neg(\Diamond P \to Q)$				(3)
5.	$\langle 1,2\rangle\Diamond P$				(4)
6.	$\langle 1,2\rangle\neg Q$				(4)
7.	$\langle 1,2,3\rangle P$				(5)
8.	$\langle 1,2\rangle\Box(P \to \Box Q)$				(2)
9.	$\langle 1,2,3\rangle P \to \Box Q$				(8)
10.	$\langle 1,2,3\rangle\neg P$	(9)	11.	$\langle 1,2,3\rangle\Box Q$	(9)
	$\otimes$		12.	$\langle 1,2\rangle Q$	(11) (symmetrisyys)
				$\otimes$	

d)

1.	$\langle 1\rangle\neg\big(\Diamond\Box(P\vee\Box Q)\to\Diamond\Box\Diamond(\Diamond\neg Q\to P)\big)$		
2.	$\langle 1\rangle\Diamond\Box(P\vee\Box Q)$		(1)
3.	$\langle 1\rangle\neg\Diamond\Box\Diamond(\Diamond\neg Q\to P)$		(1)
4.	$\langle 1,2\rangle\Box(P\vee\Box Q)$		(2)
5.	$\langle 1,2\rangle\neg\Box\Diamond(\Diamond\neg Q\to P)$		(3)
6.	$\langle 1,2,3\rangle\neg\Diamond(\Diamond\neg Q\to P)$		(5)
7.	$\langle 1,2,3,4\rangle\neg(\Diamond\neg Q\to P)$		(6) (sarjallisuus)
8.	$\langle 1,2,3,4\rangle\Diamond\neg Q$		(7)
9.	$\langle 1,2,3,4\rangle\neg P$		(7)
10.	$\langle 1,2,3,4\rangle P\vee\Box Q$		(4) (transitiivisuus)
11.	$\langle 1,2,3,4\rangle P$ (10)	12.	$\langle 1,2,3,4\rangle\Box Q$ (10)
	$\otimes$	13.	$\langle 1,2,3,4,5\rangle\neg Q$ (8)
		14.	$\langle 1,2,3,4,5\rangle Q$ (12)
			$\otimes$

e)

1.	$1\ \neg(\Box\Diamond\neg\Box P\to\neg\Box P)$	
2.	$1\ \Box\Diamond\neg\Box P$	(1)
3.	$1\ \neg\neg\Box P$	(1)
4.	$1\ \Diamond\neg\Box P$	(2)
5.	$2\ \neg\Box P$	(4)
6.	$3\ \neg P$	(5)
7.	$1\ \Box P$	(3)
8.	$3\ P$	(7)
	$\otimes$	

2. Systemaattinen **K**-taulu:

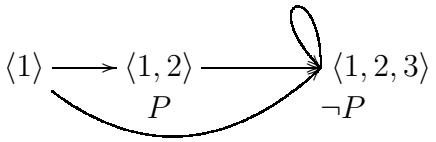
1.	$\langle 1\rangle\neg(\Diamond P\to\Diamond\Box P)$	
2.	$\langle 1\rangle\Diamond P$	(1)
3.	$\langle 1\rangle\neg\Diamond\Box P$	(1)
4.	$\langle 1,2\rangle P$	(2)
5.	$\langle 1,2\rangle\neg\Box P$	(3)
6.	$\langle 1\rangle\neg\Diamond\Box P$	(3)
7.	$\langle 1,2,3\rangle\neg P$	(5)

Ei **K**-pätevä. $\langle 1\rangle \longrightarrow \langle 1,2\rangle \longrightarrow \langle 1,2,3\rangle$ (P on $\langle 1,2\rangle$:ssa, $\neg P$ on $\langle 1,2,3\rangle$:ssa)

Systemaattinen **K4**-taulu:

1.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Box P)$	
2.	$\langle 1\rangle\Diamond P$	(1)
3.	$\langle 1\rangle\neg\Diamond\Box P$	(1)
4.	$\langle 1,2\rangle P$	(2)
5.	$\langle 1,2\rangle\neg\Box P$	(3)
6.	$\langle 1\rangle\neg\Diamond\Box P$	(3)
7.	$\langle 1,2,3\rangle\neg P$	(5)
8.	$\langle 1,2,3\rangle\neg\Box P$	(6) (transitiivisuus)
9.	$\langle 1\rangle\neg\Diamond\Box P$	(6)
10.	$\langle 1,2,3,4\rangle\neg P$	(8)
	$\vdots$	

Ei **K4**-pätevä.



3.

1.	$\langle 1\rangle\neg\big((\Diamond P \to \Diamond\Diamond P) \wedge \neg P\big)$				
2.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$	(1)	3.	$\langle 1\rangle\neg\neg P$	(1)
6.	$\langle 1\rangle\Diamond P$	(2)	4.	$\langle 1\rangle P$	(3)
7.	$\langle 1\rangle\neg\Diamond\Diamond P$	(2)	5.	$\langle 1\rangle\neg P$	(LP)
8.	$\langle 1,2\rangle P$	(6)		$\otimes$	
9.	$\langle 1,2\rangle\neg\Diamond P$	(7)			
10.	$\langle 1,2\rangle\Box\neg P \to \neg P$	(GP)			
11.	$\langle 1,2\rangle\neg\Box\neg P$	(10)	12.	$\langle 1,2\rangle\neg P$	(10)
13.	$\langle 1,2,3\rangle\neg\neg P$	(11)		$\otimes$	
14.	$\langle 1,2,3\rangle P$	(12)			
15.	$\langle 1,2,3\rangle\neg P$	(9)			