T-79.146 **Kevät 2002**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 6
Ratkaisut

1. a)

1.	$\langle 1\rangle\neg\big(\Box P \to \Box(Q \to P)\big)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box(Q \to P)$	(1)
4.	$\langle 1,2\rangle\neg(Q \to P)$	(3)
5.	$\langle 1,2\rangle Q$	(4)
6.	$\langle 1,2\rangle\neg P$	(4)
7.	$\langle 1,2\rangle P$	(2)
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Box(P \to Q) \to (\neg\Box\neg P \to \neg\Box\neg Q)\big)$				
2.	$\langle 1\rangle\Box(P \to Q)$	(1)			
3.	$\langle 1\rangle\neg(\neg\Box\neg P \to \neg\Box\neg Q)$	(1)			
4.	$\langle 1\rangle\neg\Box\neg P$	(3)			
5.	$\langle 1\rangle\neg\neg\Box\neg Q$	(3)			
6.	$\langle 1\rangle\Box\neg Q$	(5)			
7.	$\langle 1,2\rangle\neg\neg P$	(4)			
8.	$\langle 1,2\rangle P$	(7)			
9.	$\langle 1,2\rangle\neg Q$	(6)			
10.	$\langle 1,2\rangle P \to Q$	(2)			
11.	$\langle 1,2\rangle\neg P$	(10)	12.	$\langle 1,2\rangle Q$	(10)
	$\otimes$			$\otimes$	

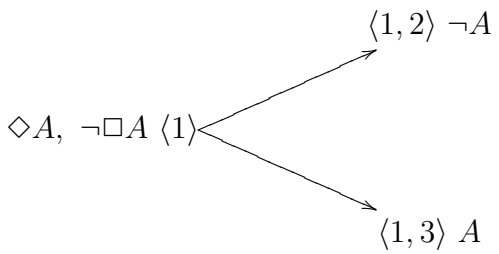
c)

1.	$\langle 1\rangle\neg\big((\Box P \wedge \Box Q) \to \Box(P \wedge Q)\big)$				
2.	$\langle 1\rangle\Box P \wedge \Box Q$	(1)			
3.	$\langle 1\rangle\neg\Box(P \wedge Q)$	(1)			
4.	$\langle 1\rangle\Box P$	(2)			
5.	$\langle 1\rangle\Box Q$	(2)			
6.	$\langle 1,2\rangle\neg(P \wedge Q)$	(3)			
7.	$\langle 1,2\rangle P$	(4)			
8.	$\langle 1,2\rangle Q$	(5)			
9.	$\langle 1,2\rangle\neg P$	(6)	10.	$\langle 1,2\rangle\neg Q$	(6)
	$\otimes$			$\otimes$	

2. a)

1.	$\langle 1\rangle\neg(\Diamond A \to \Box A)$	
2.	$\langle 1\rangle\Diamond A$	(1)
3.	$\langle 1\rangle\neg\Box A$	(1)
4.	$\langle 1\rangle\neg\Box\neg A$	(2)
5.	$\langle 1,2\rangle\neg A$	(3)
6.	$\langle 1,3\rangle\neg\neg A$	(4)
7.	$\langle 1,3\rangle A$	(6)

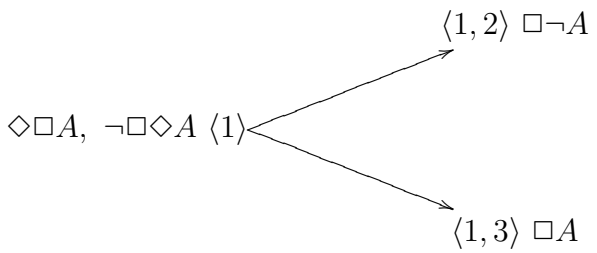
Ei **K**-pätevä.



b)

1.	$\langle 1\rangle\neg(\Diamond\Box A \to \Box\Diamond A)$	
2.	$\langle 1\rangle\Diamond\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Diamond A$	(1)
4.	$\langle 1\rangle\neg\Box\neg\Box A$	(2)
5.	$\langle 1,2\rangle\neg\Diamond A$	(3)
6.	$\langle 1,3\rangle\neg\neg\Box A$	(4)
7.	$\langle 1,2\rangle\neg\neg\Box\neg A$	(5)
8.	$\langle 1,3\rangle\Box A$	(6)
9.	$\langle 1,2\rangle\Box\neg A$	(7)

Ei **K**-pätevä.



c)

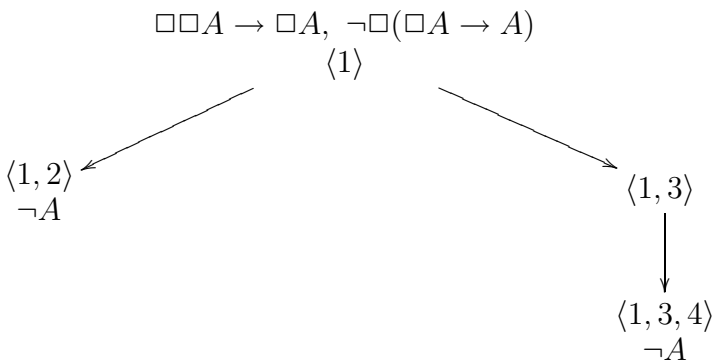
1.	$\langle 1\rangle\neg(\Box\Box A \to \Box\Box\Box A)$	
2.	$\langle 1\rangle\Box\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Box\Box A$	(1)
4.	$\langle 1,2\rangle\neg\Box\Box A$	(3)
5.	$\langle 1,2\rangle\Box A$	(2)
6.	$\langle 1,2,3\rangle\neg\Box A$	(4)
7.	$\langle 1,2,3\rangle A$	(5)
8.	$\langle 1,2,3,4\rangle\neg A$	(6)

Ei **K**-pätevä. $\langle 1\rangle \xrightarrow{\quad} \langle 1,2\rangle \xrightarrow{\quad} \langle 1,2,3\rangle \xrightarrow{A} \langle 1,2,3,4\rangle$ ($\neg A$)

d)

1.	$\langle 1\rangle\neg\big((\Box\Box A \to \Box A) \to \Box(\Box A \to A)\big)$				
2.	$\langle 1\rangle\Box\Box A \to \Box A$	(1)			
3.	$\langle 1\rangle\neg\Box(\Box A \to A)$	(1)			
4.	$\langle 1\rangle\neg\Box\Box A$	(2)	5.	$\langle 1\rangle\Box A$	(2)
6.	$\langle 1,2\rangle\neg(\Box A \to A)$	(3)	7.	$\langle 1,2\rangle\neg(\Box A \to A)$	(3)
8.	$\langle 1,3\rangle\neg\Box A$	(4)	12.	$\langle 1,2\rangle\Box A$	(7)
9.	$\langle 1,2\rangle\Box A$	(6)	13.	$\langle 1,2\rangle\neg A$	(7)
10.	$\langle 1,2\rangle\neg A$	(6)	14.	$\langle 1,2\rangle A$	(5)
11.	$\langle 1,3,4\rangle\neg A$	(8)		$\otimes$	

Ei **K**-pätevä.



3. a)

1.	$\langle 1\rangle\neg\Big(\big(\Box(P\to Q)\to\Box(Q\to R)\big)\to\Box(P\to R)\Big)$							
2.	$\langle 1\rangle\Box(P\to Q)\to\Box(Q\to R)$					(1)		
3.	$\langle 1\rangle\neg\Box(P\to R)$					(1)		
4.	$\langle 1\rangle\neg\Box(P\to Q)$	(2)	5.	$\langle 1\rangle\Box(Q\to R)$	(2)			
6.	$\langle 1,2\rangle\neg(P\to R)$	(3)	7.	$\langle 1,2\rangle\neg(P\to R)$	(3)			
8.	$\langle 1,3\rangle\neg(P\to Q)$	(4)	13.	$\langle 1,2\rangle Q\to R$	(5)			
9.	$\langle 1,2\rangle P$	(6)	14.	$\langle 1,2\rangle P$	(7)			
10.	$\langle 1,2\rangle\neg R$	(6)	15.	$\langle 1,2\rangle\neg R$	(7)			
11.	$\langle 1,3\rangle P$	(8)	16.	$\langle 1,2\rangle\neg Q$	(13)	17.	$\langle 1,2\rangle R$	(13)
12.	$\langle 1,3\rangle\neg Q$	(8)					$\otimes$	

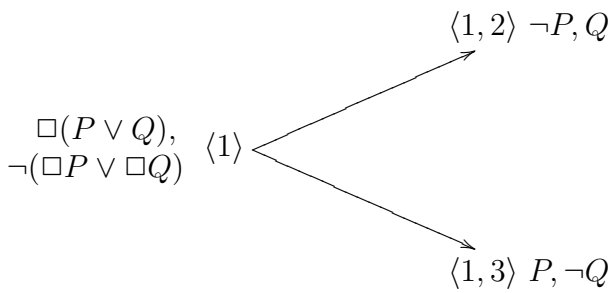
Lause ei ole **K**-pätevä. Malli lauseen negaatiolle voidaan muodostaa taulun minkä tahansa avoimen haaran perusteella, esim.

$$\Box(P\to Q)\to\Box(Q\to R),\neg\Box(P\to R)$$
$$\langle 1\rangle$$
$$\downarrow$$
$$\langle 1,2\rangle$$
$$P,\neg Q,\neg R$$

b)

1.	$\langle 1\rangle\neg\big((\Diamond(P\vee Q)\to(\Diamond P\vee\Diamond Q)\big)$							
2.	$\langle 1\rangle\Diamond(P\vee Q)$					(1)		
3.	$\langle 1\rangle\neg(\Diamond P\vee\Diamond Q)$					(1)		
4.	$\langle 1\rangle\neg\Diamond P$					(3)		
5.	$\langle 1\rangle\neg\Diamond Q$					(3)		
6.	$\langle 1\rangle\neg\neg\Box\neg P$					(4)		
7.	$\langle 1\rangle\Box\neg P$					(6)		
8.	$\langle 1\rangle\neg\neg\Box\neg Q$					(5)		
9.	$\langle 1\rangle\Box\neg Q$					(8)		
10.	$\langle 1\rangle\neg\Box\neg(P\vee Q)$					(2)		
11.	$\langle 1,2\rangle\neg\neg(P\vee Q)$					(10)		
12.	$\langle 1,2\rangle P\vee Q$					(11)		
13.	$\langle 1,2\rangle\neg P$					(7)		
14.	$\langle 1,2\rangle\neg Q$					(9)		
15.	$\langle 1,2\rangle P$	(12)	16.	$\langle 1,2\rangle Q$	(12)			
	$\otimes$			$\otimes$				

c)

1.	$\langle 1\rangle\neg\big(\Box(P\vee Q)\to(\Box P\vee\Box Q)\big)$				
2.	$\langle 1\rangle\Box(P\vee Q)$	(1)			
3.	$\langle 1\rangle\neg(\Box P\vee\Box Q)$	(1)			
4.	$\langle 1\rangle\neg\Box P$	(3)			
5.	$\langle 1\rangle\neg\Box Q$	(3)			
6.	$\langle 1,2\rangle\neg P$	(4)			
7.	$\langle 1,2\rangle P\vee Q$	(2)			
8.	$\langle 1,2\rangle P$ (7)	9.	$\langle 1,2\rangle Q$	(7)	
	$\otimes$	10.	$\langle 1,3\rangle\neg Q$	(5)	
		11.	$\langle 1,3\rangle P\vee Q$	(2)	
		12.	$\langle 1,3\rangle P$ (11)	13.	$\langle 1,3\rangle Q$ (11)
					$\otimes$

Ei **K**-pätevä.