T-79.146 **Spring 2002/IN**
Logic in Computer Science: Special Topics I
Exam 8.5.2002

1. a) Given a set of models C and formula P does it hold that if P is valid in the set of model C, then $\neg P$ is also valid in C. Provide a short argument justifying your answer.

b) Define what it means that a given logic has the finite model property and that it is decidable.

c) Construct the positive normal form of the CTL formula $(\mathbf{EXAFE}(P\mathbf{U}Q)) \rightarrow P$.

2. Use the tableau method to determine whether the statements below hold. Construct a (counter) model whenever possible.

a) $\{\} \models_{\mathbf{K4}} \{\neg\Diamond P\} \Longrightarrow \Box(\Box P \vee \neg\Diamond P)$, where **K4** is the class of transitive frames.

b) $\{\neg\Box\neg P\} \models_{\mathbf{K}} \{\Box\Box(P \rightarrow Q)\} \Longrightarrow \neg\Box\neg Q$, where **K** is the class of all frames.

c) For all transitive and symmetric models it holds that if formula $\Box(Q \rightarrow \Box P)$ is true in some state of the model, then formula $\Diamond\Box(\Box P \rightarrow Q)$ is false in that state.

P and Q are atomic formulae.

3. Consider a Hilbert proof system whose axioms are all classical tautologies and all formulae of the form $\Box P \rightarrow Q$ and $\neg\Box P \rightarrow \Box P$ and where Modus Ponens and Necessitation are used as the inference rules.

a) Let **L** be the class of all symmetric frames. Define what it means that a Hilbert proof system is sound and complete for the frame logic **L**.

b) Show that the proof system above is not sound for the frame logic **L**.

c) Show that the proof system above is complete for the frame logic **L**.

4. a) Formalize the following conditions in CTL.

(i) A state where condition `req` holds occurs infinitely often on any computation path.

(ii) For any computation path condition `busy` holds until condition `done` starts to hold.

b) Let $\langle S, R, v\rangle$ be a (CTL) model for a distributed system where $S = \{a, b, c, d, e\}$, $R = \{\langle a,b\rangle, \langle a,c\rangle, \langle c,a\rangle, \langle c,b\rangle, \langle b,d\rangle, \langle d,d\rangle, \langle c,e\rangle, \langle e,e\rangle\}$, $v(d,\texttt{ack}) = v(e,\texttt{ack}) = \text{true}$, $v(a,\texttt{ack}) = v(b,\texttt{ack}) = v(c,\texttt{ack}) = \text{false}$, $v(a,\texttt{req}) = v(d,\texttt{req}) = \text{true}$, $v(b,\texttt{req}) = v(c,\texttt{req}) = v(e,\texttt{req}) = \text{false}$.

Determine those states of the model where the formula $\neg\mathbf{E}(\top\mathbf{U}\neg(\neg\texttt{req} \vee \mathbf{A}(\top\mathbf{U}\texttt{ack})))$ is true.

c) Determine whether the CTL formula

$$\mathbf{EG}P \rightarrow \mathbf{EXEG}P$$

is valid using the tableau method.

Properties of relation R:
Reflexivity: $\forall s(sRs)$
Symmetry: $\forall s\forall t(sRt \rightarrow tRs)$
Transitivity: $\forall s\forall t\forall u(sRt \wedge tRu \rightarrow sRu)$