Tik-79.146 **Kevät 2001**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 6
Ratkaisut

Aksioomat ja päättelysäännöt:

$$
\begin{array}{rl}
\text{K:} & \Box(P \to Q) \to (\Box P \to \Box Q) \\
\text{T:} & \Box P \to P \\
\text{D:} & \Box P \to \Diamond P \quad \text{tai} \quad \Box P \to \neg\Box\neg P \\
\text{4:} & \Box P \to \Box\Box P \\
\text{5:} & \neg\Box P \to \Box\neg\Box P \\
\text{MP:} & \dfrac{P, P \to Q}{Q} \\
\text{N:} & \dfrac{P}{\Box P}
\end{array}
$$

1. a)

$$
\begin{array}{lll}
1. & P \to (Q \to P) & [\text{Tautologia}] \\
2. & \Box\big(P \to (Q \to P)\big) & [\text{N, 1}] \\
3. & \Box\big(P \to (Q \to P)\big) \to \big(\Box P \to \Box(Q \to P)\big) & [\text{K}] \\
4. & \Box P \to \Box(Q \to P) & [\text{MP, 2, 3}]
\end{array}
$$

b)

$$
\begin{array}{lll}
1. & \Box(P \to Q) & [\text{GP}] \\
2. & (P \to Q) \to (\neg Q \to \neg P) & [\text{Tautologia}] \\
3. & \Box\big((P \to Q) \to (\neg Q \to \neg P)\big) & [\text{N, 2}] \\
4. & \Box\big((P \to Q) \to (\neg Q \to \neg P)\big) \to & \\
 & \quad \big(\Box(P \to Q) \to \Box(\neg Q \to \neg P)\big) & [\text{K}] \\
5. & \Box(P \to Q) \to \Box(\neg Q \to \neg P) & [\text{MP, 3, 4}] \\
6. & \Box(\neg Q \to \neg P) & [\text{MP, 1, 5}] \\
7. & \Box(\neg Q \to \neg P) \to (\Box\neg Q \to \Box\neg P) & [\text{K}] \\
8. & \Box\neg Q \to \Box\neg P & [\text{MP, 6, 7}]
\end{array}
$$

2. a)

$$
\begin{array}{lll}
1. & \neg\Box P \to \Box\neg\Box P & [\text{5-aksiooma}] \\
2. & (\neg\Box P \to \Box\neg\Box P) \to (\neg\Box\neg\Box P \to \Box P) & [\text{Tautologia}] \\
3. & \neg\Box\neg\Box P \to \Box P & [\text{MP, 1, 2}] \\
4. & \Box\Box P & [\text{GP}] \\
5. & \Box\Box P \to \neg\Box\neg\Box P & [\text{D}] \\
6. & \neg\Box\neg\Box P & [\text{MP, 4, 5}] \\
7. & \Box P & [\text{MP, 6, 3}]
\end{array}
$$

b) **S4** on **KT4**.

$$
\begin{array}{lll}
1. & \neg\bot & [\text{Tautologia}] \\
2. & \Box\bot \to \bot & [\text{T}] \\
3. & (\Box\bot \to \bot) \to (\neg\bot \to \neg\Box\bot) & [\text{Tautologia}] \\
4. & \neg\bot \to \neg\Box\bot & [\text{MP, 2, 3}] \\
5. & \neg\Box\bot & [\text{MP, 1, 4}]
\end{array}
$$

3.

$$
\begin{array}{lll}
1. & P \to Q & [\text{GP}] \\
2. & \neg Q \to P & [\text{GP}] \\
3. & (P \to Q) \to \big((\neg Q \to P) \to Q\big) & [\text{Tautologia}] \\
4. & (\neg Q \to P) \to Q & [\text{MP, 1, 3}] \\
5. & Q & [\text{MP, 2, 4}] \\
6. & \Box Q & [\text{N, 5}] \\
\hline
7. & Q \to S & [\text{LP}] \\
8. & S & [\text{MP, 5, 7}] \\
9. & \Box Q \to (S \to \Box Q \wedge S) & [\text{Tautologia}] \\
10. & S \to \Box Q \wedge S & [\text{MP, 6, 9}] \\
11. & \Box Q \wedge S & [\text{MP, 8, 10}]
\end{array}
$$