Tik-79.146 **Kevät 1999**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 7
Ratkaisut

1. a)

1.	$\langle 1\rangle\neg\big(\Box P \to \Box(Q \to P)\big)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box(Q \to P)$	(1)
4.	$\langle 1,2\rangle\neg(Q \to P)$	(3)
5.	$\langle 1,2\rangle Q$	(4)
6.	$\langle 1,2\rangle\neg P$	(4)
7.	$\langle 1,2\rangle P$	(2)
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Box(P \to Q) \to (\neg\Box\neg P \to \neg\Box\neg Q)\big)$	
2.	$\langle 1\rangle\Box(P \to Q)$	(1)
3.	$\langle 1\rangle\neg(\neg\Box\neg P \to \neg\Box\neg Q)$	(1)
4.	$\langle 1\rangle\neg\Box\neg P$	(3)
5.	$\langle 1\rangle\neg\neg\Box\neg Q$	(3)
6.	$\langle 1\rangle\Box\neg Q$	(5)
7.	$\langle 1,2\rangle\neg\neg P$	(4)
8.	$\langle 1,2\rangle P$	(7)
9.	$\langle 1,2\rangle\neg Q$	(6)
10.	$\langle 1,2\rangle(P \to Q)$	(2)
11.	$\langle 1,2\rangle\neg P$ (10) $\otimes$	12. $\langle 1,2\rangle Q$ (10) $\otimes$

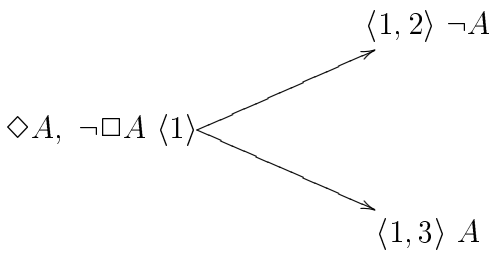
c)

1.	$\langle 1\rangle\neg\big((\Box P \wedge \Box Q) \to \Box(P \wedge Q)\big)$	
2.	$\langle 1\rangle\Box P \wedge \Box Q$	(1)
3.	$\langle 1\rangle\neg\Box(P \wedge Q)$	(1)
4.	$\langle 1\rangle\Box P$	(2)
5.	$\langle 1\rangle\Box Q$	(2)
6.	$\langle 1,2\rangle\neg(P \wedge Q)$	(3)
7.	$\langle 1,2\rangle P$	(4)
8.	$\langle 1,2\rangle Q$	(5)
9.	$\langle 1,2\rangle\neg P$ (6) $\otimes$	10. $\langle 1,2\rangle\neg Q$ (6) $\otimes$

2. a)

1.	$\langle 1\rangle\neg(\Diamond A \to \Box A)$	
2.	$\langle 1\rangle\Diamond A$	(1)
3.	$\langle 1\rangle\neg\Box A$	(1)
4.	$\langle 1\rangle\neg\Box\neg A$	(2)
5.	$\langle 1,2\rangle\neg A$	(3)
6.	$\langle 1,3\rangle\neg\neg A$	(4)
7.	$\langle 1,3\rangle A$	(6)

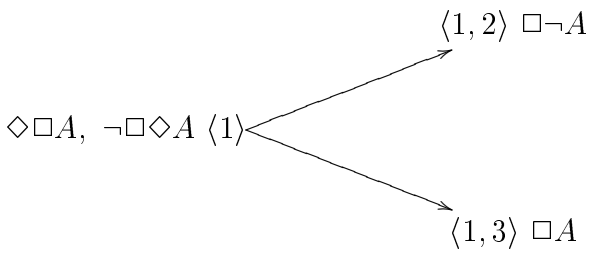
Ei **K**-pätevä.



b)

1.	$\langle 1\rangle\neg(\Diamond\Box A \to \Box\Diamond A)$	
2.	$\langle 1\rangle\Diamond\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Diamond A$	(1)
4.	$\langle 1\rangle\neg\Box\neg\Box A$	(2)
5.	$\langle 1,2\rangle\neg\Diamond A$	(3)
6.	$\langle 1,3\rangle\neg\neg\Box A$	(4)
7.	$\langle 1,2\rangle\neg\neg\Box\neg A$	(5)
8.	$\langle 1,3\rangle\Box A$	(6)
9.	$\langle 1,2\rangle\Box\neg A$	(7)

Ei **K**-pätevä.



c)

1.	$\langle 1\rangle\neg(\Box\Box A \to \Box\Box\Box A)$	
2.	$\langle 1\rangle\Box\Box A$	(1)
3.	$\langle 1\rangle\neg\Box\Box\Box A$	(1)
4.	$\langle 1,2\rangle\neg\Box\Box A$	(3)
5.	$\langle 1,2\rangle\Box A$	(2)
6.	$\langle 1,2,3\rangle\neg\Box A$	(4)
7.	$\langle 1,2,3\rangle A$	(5)
8.	$\langle 1,2,3,4\rangle\neg A$	(6)

Ei **K**-pätevä. $\langle 1\rangle \longrightarrow \langle 1,2\rangle \longrightarrow \underset{\langle 1,2,3\rangle}{A} \longrightarrow \underset{\langle 1,2,3,4\rangle}{\neg A}$

3. a)

1.	$\langle 1\rangle\neg(\Box P \to \Box\Box\Box\Box P)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box\Box\Box\Box P$	(1)
4.	$\langle 1,2\rangle\neg\Box\Box\Box P$	(3)
5.	$\langle 1,2,3\rangle\neg\Box\Box P$	(4)
6.	$\langle 1,2,3,4\rangle\neg\Box P$	(5)
7.	$\langle 1,2,3,4,5\rangle\neg P$	(6)
8.	$\langle 1,2,3,4,5\rangle P$	(2) transitiivinen
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Diamond\neg P \to (\Diamond P \to \Diamond\Diamond P)\big)$	
2.	$\langle 1\rangle\Diamond\neg P$	(1)
3.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$	(1)
4.	$\langle 1\rangle\Diamond P$	(3)
5.	$\langle 1\rangle\neg\Diamond\Diamond P$	(3)
6.	$\langle 1,2\rangle P$	(4)
7.	$\langle 1,2\rangle\neg\Diamond P$	(5)
8.	$\langle 1,2\rangle\neg P$	(7) refleksiivinen
	$\otimes$	

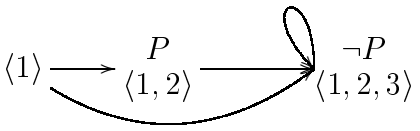
c)

1.	$\langle 1\rangle\neg\big(\Box\Box(P\to\Box Q)\to\Box(\Diamond P\to Q)\big)$		
2.	$\langle 1\rangle\Box\Box(P\to\Box Q)$		(1)
3.	$\langle 1\rangle\neg\Box(\Diamond P\to Q)$		(1)
4.	$\langle 1,2\rangle\neg(\Diamond P\to Q)$		(3)
5.	$\langle 1,2\rangle\Diamond P$		(4)
6.	$\langle 1,2\rangle\neg Q$		(4)
7.	$\langle 1,2,3\rangle P$		(5)
8.	$\langle 1,2\rangle\Box(P\to\Box Q)$		(2)
9.	$\langle 1,2,3\rangle P\to\Box Q$		(8)
10.	$\langle 1,2,3\rangle\neg P$ (9)	11.	$\langle 1,2,3\rangle\Box Q$ (9)
	$\otimes$	12.	$\langle 1,2\rangle Q$ (11) symmetrinen
			$\otimes$

4. Systemaattinen **K4**-taulu:

1.	$\langle 1\rangle\neg(\Diamond P\to\Diamond\Box P)$	
2.	$\langle 1\rangle\Diamond P$	(1)
3.	$\langle 1\rangle\neg\Diamond\Box P$	(1)
4.	$\langle 1,2\rangle P$	(2)
5.	$\langle 1,2\rangle\neg\Box P$	(3)
6.	$\langle 1\rangle\neg\Diamond\Box P$	(3)
7.	$\langle 1,2,3\rangle\neg P$	(5)
8.	$\langle 1,2,3\rangle\neg\Box P$	(6) transitiivinen
9.	$\langle 1\rangle\neg\Diamond\Box P$	(6)
10.	$\langle 1,2,3,4\rangle\neg P$	(8)
	$\vdots$	

Ei **K4**-pätevä, eikä **K**-pätevä.



5.

1.	1,	$\neg(\Box\Diamond\neg\Box P\to\neg\Box P)$	
2.	1,	$\Box\Diamond\neg\Box P$	(1)
3.	1,	$\neg\neg\Box P$	(1)
4.	1,	$\Diamond\neg\Box P$	(2)
5.	2,	$\neg\Box P$	(4)
6.	3,	$\neg P$	(5)
7.	1,	$\Box P$	(3)
8.	3,	P	(7)
		$\otimes$	

6.

1.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$				
2.	$\langle 1\rangle\Diamond P$	(1)			
3.	$\langle 1\rangle\neg\Diamond\Diamond P$	(1)			
4.	$\langle 1,2\rangle P$	(2)			
5.	$\langle 1,2\rangle\neg\Diamond P$	(3)			
6.	$\langle 1,2\rangle\Box\neg P \to \neg P$	(GP)			
7.	$\langle 1,2\rangle\neg\Box\neg P$	(6)	8.	$\langle 1,2\rangle\neg P$	(6)
9.	$\langle 1,2,3\rangle\neg\neg P$	(7)		$\otimes$	
10.	$\langle 1,2,3\rangle P$	(9)			
11.	$\langle 1,2,3\rangle\neg P$	(5)			
	$\otimes$				