Tik-79.146 **Kevät 1998**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 9
Ratkaisut

1. a)

1.	$\langle 1\rangle\neg(\Box P \to \Box\Box\Box\Box P)$	
2.	$\langle 1\rangle\Box P$	(1)
3.	$\langle 1\rangle\neg\Box\Box\Box\Box P$	(1)
4.	$\langle 1,2\rangle\neg\Box\Box\Box P$	(3)
5.	$\langle 1,2,3\rangle\neg\Box\Box P$	(4)
6.	$\langle 1,2,3,4\rangle\neg\Box P$	(5)
7.	$\langle 1,2,3,4,5\rangle\neg P$	(6)
8.	$\langle 1,2,3,4,5\rangle P$	(2) transitiivinen
	$\otimes$	

b)

1.	$\langle 1\rangle\neg\big(\Diamond\neg P \to (\Diamond P \to \Diamond\Diamond P)\big)$	
2.	$\langle 1\rangle\Diamond\neg P$	(1)
3.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$	(1)
4.	$\langle 1\rangle\Diamond P$	(3)
5.	$\langle 1\rangle\neg\Diamond\Diamond P$	(3)
6.	$\langle 1,2\rangle P$	(4)
7.	$\langle 1,2\rangle\neg\Diamond P$	(5)
8.	$\langle 1,2\rangle\neg P$	(7) refleksiivinen
	$\otimes$	

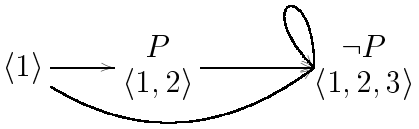
c)

1.	$\langle 1\rangle\neg\big(\Box\Box(P \to \Box Q) \to \Box(\Diamond P \to Q)\big)$			
2.	$\langle 1\rangle\Box\Box(P \to \Box Q)$			(1)
3.	$\langle 1\rangle\neg\Box(\Diamond P \to Q)$			(1)
4.	$\langle 1,2\rangle\neg(\Diamond P \to Q)$			(3)
5.	$\langle 1,2\rangle\Diamond P$			(4)
6.	$\langle 1,2\rangle\neg Q$			(4)
7.	$\langle 1,2,3\rangle P$			(5)
8.	$\langle 1,2\rangle\Box(P \to \Box Q)$			(2)
9.	$\langle 1,2,3\rangle P \to \Box Q$			(8)
10.	$\langle 1,2,3\rangle\neg P$	(9)	11. $\langle 1,2,3\rangle\Box Q$	(9)
	$\otimes$		12. $\langle 1,2\rangle Q$	(11) symmetrinen
			$\otimes$	

2. Systemaattinen **K4**-taulu:

1.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Box P)$	
2.	$\langle 1\rangle\Diamond P$	(1)
3.	$\langle 1\rangle\neg\Diamond\Box P$	(1)
4.	$\langle 1,2\rangle P$	(2)
5.	$\langle 1,2\rangle\neg\Box P$	(3)
6.	$\langle 1\rangle\neg\Diamond\Box P$	(3)
7.	$\langle 1,2,3\rangle\neg P$	(5)
8.	$\langle 1,2,3\rangle\neg\Box P$	(6) transitiivinen
9.	$\langle 1\rangle\neg\Diamond\Box P$	(6)
10.	$\langle 1,2,3,4\rangle\neg P$	(8)
	$\vdots$	

Ei **K4**-pätevä, eikä **K**-pätevä.



3.

1.	$1,\ \neg(\Box\Diamond\neg\Box P \to \neg\Box P)$	
2.	$1,\ \Box\Diamond\neg\Box P$	(1)
3.	$1,\ \neg\neg\Box P$	(1)
4.	$1,\ \Diamond\neg\Box P$	(2)
5.	$2,\ \neg\Box P$	(4)
6.	$3,\ \neg P$	(5)
7.	$1,\ \Box P$	(3)
8.	$3,\ P$	(7)
	$\otimes$	

4.

1.	$\langle 1\rangle\neg(\Diamond P \to \Diamond\Diamond P)$				
2.	$\langle 1\rangle\Diamond P$	(1)			
3.	$\langle 1\rangle\neg\Diamond\Diamond P$	(1)			
4.	$\langle 1,2\rangle P$	(2)			
5.	$\langle 1,2\rangle\neg\Diamond P$	(3)			
6.	$\langle 1,2\rangle\Box\neg P \to \neg P$	(GP)			
7.	$\langle 1,2\rangle\neg\Box\neg P$	(6)	8.	$\langle 1,2\rangle\neg P$	(6)
9.	$\langle 1,2,3\rangle\neg\neg P$	(7)		$\otimes$	
10.	$\langle 1,2,3\rangle P$	(9)			
11.	$\langle 1,2,3\rangle\neg P$	(5)			
	$\otimes$				