Tik-79.146 **Kevät 1998**
Logiikka tietotekniikassa: erityiskysymyksiä I
Laskuharjoitus 6
Ratkaisut

Aksioomat ja päättelysäännöt:

$$\text{K:}\quad \Box(P \to Q) \to (\Box P \to \Box Q)$$

$$\text{MP:}\quad \frac{P,\ P \to Q}{Q}$$

$$\text{N:}\quad \frac{P}{\Box P}$$

1. a)

1.	$P \to (Q \to P)$	[Tautologia]
2.	$\Box\big(P \to (Q \to P)\big)$	[N, 1]
3.	$\Box\big(P \to (Q \to P)\big) \to \big(\Box P \to \Box(Q \to P)\big)$	[K]
4.	$\Box P \to \Box(Q \to P)$	[MP, 2,3]

b)

1.	$\Box(P \to Q)$	[GP]
2.	$(P \to Q) \to (\neg Q \to \neg P)$	[Taut]
3.	$\Box\big((P \to Q) \to (\neg Q \to \neg P)\big)$	[N, 2]
4.	$\Box\big((P \to Q) \to (\neg Q \to \neg P)\big) \to$ $\big(\Box(P \to Q) \to \Box(\neg Q \to \neg P)\big)$	[K]
5.	$\Box(P \to Q) \to \Box(\neg Q \to \neg P)$	[MP, 3,4]
6.	$\Box(\neg Q \to \neg P)$	[MP, 1,5]
7.	$\Box(\neg Q \to \neg P) \to (\Box\neg Q \to \Box\neg P)$	[K]
8.	$\Box\neg Q \to \Box\neg P$	[MP, 6,7]
9.	$(\Box\neg Q \to \Box\neg P) \to (\neg\Box\neg P \to \neg\Box\neg Q)$	[Taut]
10.	$\neg\Box\neg P \to \neg\Box\neg Q$	[MP, 8,9]

2. a)

1.	$P \to Q$	[GP]
2.	$\neg Q \to P$	[GP]
3.	$(P \to Q) \to \big((\neg Q \to P) \to Q\big)$	[Taut]
4.	$(\neg Q \to P) \to Q$	[MP, 1,3]
5.	Q	[MP, 2,4]
6.	$\Box Q$	[N, 5]
7.	$\neg Q \vee S$	[LP]
8.	$(\neg Q \vee S) \to (Q \to S)$	[Taut]
9.	$Q \to S$	[MP, 7,8]
10.	S	[MP, 5,9]
11.	$\Box Q \to (S \to \Box Q \wedge S)$	[Taut]
12.	$S \to \Box Q \wedge S$	[MP, 6,11]
13.	$\Box Q \wedge S$	[MP, 10,12]

b)

1.	$Q \to \neg P$	[GP]
2.	$\Box(Q \to \neg P)$	[N, 1]
3.	$\Box(Q \to \neg P) \to (\Box Q \to \Box\neg P)$	[K]
4.	$\Box Q \to \Box\neg P$	[MP, 2, 3]
5.	$\Diamond Q \to \Box Q$	[GP]
6.	$(\Diamond Q \to \Box Q) \to$ $\big((\Box Q \to \Box\neg P) \to (\Diamond Q \to \Box\neg P)\big)$	[Taut]
7.	$(\Box Q \to \Box\neg P) \to (\Diamond Q \to \Box\neg P)$	[MP, 5, 6]
8.	$\Diamond Q \to \Box\neg P$	[MP, 4, 7]
9.	$(\neg\Box\neg Q \to \Box\neg P) \to (\neg\Box\neg P \to \Box\neg Q)$	[Taut]
10.	$\neg\Box\neg P \to \Box\neg Q$	[MP, 8, 9]
11.	$\Diamond P$	[LP]
12.	$\Box\neg Q$	[MP, 10, 11]