Tik-79.146 **Kevät 1998**

Logiikka tietotekniikassa: erityiskysymyksiä I

Laskuharjoitus 1

Ratkaisut

1. a)

$$\begin{array}{c}
T\,(\neg((P \to Q) \to (Q \to P))) \\
F\,((P \to Q) \to (Q \to P)) \\
T\,(P \to Q) \\
F\,(Q \to P) \\
T\,(Q) \\
F\,(P) \\
\begin{array}{cc}
\diagup & \diagdown \\
F\,(P) & T\,(Q)
\end{array}
\end{array}$$

$M = \{Q\}$

b)

$$\begin{array}{c}
T\,(((P \vee \neg R) \leftrightarrow R) \wedge (P \to Q)) \\
T\,((P \vee \neg R) \leftrightarrow R) \\
T\,(P \to Q) \\
\begin{array}{cc}
\diagup & \diagdown \\
\begin{array}{c}
T\,(P \vee \neg R) \\
T\,(R) \\
\begin{array}{cc}
\diagup & \diagdown \\
\begin{array}{c}
T\,(P) \\
\begin{array}{cc}
\diagup & \diagdown \\
\begin{array}{c} F\,(P) \\ \otimes \end{array} & T\,(Q)
\end{array}
\end{array} &
\begin{array}{c}
T\,(\neg R) \\
F\,(R) \\
\otimes
\end{array}
\end{array}
\end{array} &
\begin{array}{c}
F\,(P \vee \neg R) \\
F\,(R) \\
F\,(P) \\
F\,(\neg R) \\
T\,(R) \\
\otimes
\end{array}
\end{array}
\end{array}$$

$M = \{P, Q, R\}$

2.

$$
\begin{array}{c}
{}^{1}T\,(Q \to P) \\
{}^{3}T\,(R \to (P \wedge Q)) \\
{}^{2}T\,(P \to (Q \wedge R)) \\
F\,(\neg Q) \\
T\,(Q) \\
\begin{array}{cc}
\begin{array}{c} {}^{1}F\,(Q) \\ \otimes \end{array} &
\begin{array}{c}
{}^{1}T\,(P) \\
\begin{array}{cc}
\begin{array}{c} {}^{2}F\,(P) \\ \otimes \end{array} &
\begin{array}{c}
{}^{2}T\,(Q \wedge R) \\
T\,(Q) \\
T\,(R) \\
\begin{array}{cc}
\begin{array}{c} {}^{3}F\,(R) \\ \otimes \end{array} &
\begin{array}{c} {}^{3}T\,(P \wedge Q) \\ T\,(P) \\ T\,(Q) \end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
\end{array}
$$

Vastamalli: $M = \{P, Q, R\}$

3. Disjunktiivinen normaalimuoto:

$$
\begin{array}{c}
T\,((P \to Q) \to (P \vee Q)) \\
\begin{array}{cc}
\begin{array}{c} F\,(P \to Q) \\ T\,(P) \\ F\,(Q) \end{array} &
\begin{array}{c}
T\,(P \vee Q) \\
\begin{array}{cc} T\,(P) & T\,(Q) \end{array}
\end{array}
\end{array}
\end{array}
$$

Avoimista haaroista saadaan DNF

$$(P \wedge \neg Q) \vee P \vee Q \equiv P \vee Q$$

Konjunktiivinen normaalimuoto:

$$
\begin{array}{c}
T\,(\neg((P \to Q) \to (P \vee Q))) \\
F\,((P \to Q) \to (P \vee Q)) \\
T\,(P \to Q) \\
F\,(P \vee Q) \\
F\,(P) \\
F\,(Q) \\
\begin{array}{cc}
F\,(P) & \begin{array}{c} T\,(Q) \\ \otimes \end{array}
\end{array}
\end{array}
$$

Avoimista haaroista saadaan DNF

$$\neg((P \to Q) \to (P \vee Q)) \equiv \neg P \wedge \neg Q$$

siis CNF on

$$(P \to Q) \to (P \vee Q) \equiv \neg(\neg P \wedge \neg Q) \equiv P \vee Q$$