Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \not\models \phi$, then $\Sigma \models \neg\phi$.

(b) Does the following hold: every sentence $\phi$ which is provable by the tableaux method is valid.

(c) Does the following hold: if $A$ is a set of atomic formulae has two most general unifiers $\theta$ and $\theta'$, then $A\theta = A\theta'$ holds necessarily.

(d) Does the following hold: propositional logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \leftrightarrow \neg C, A \leftrightarrow B \lor C\} \models B \leftrightarrow A \land \neg C$

(b) $\models \exists x(R(x) \land \neg R(f(f(x)))) \rightarrow \exists x(R(x) \land \neg R(f(x)))$

(c) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x Q(x)) \rightarrow (\forall x \neg P(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Natural numbers 0, 1, 2, ... are represented as ground terms 0, $s(0), s(s(0))$ ... built of a constant symbol 0 and a function symbol $s$ which is interpreted as the function $s(x) = x + 1$ for natural numbers $x$.

(a) Let the predicates $J2(x), J3(x)$ and $J6(x)$ mean that a natural number $x$ is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number $n$ is divisible by two and three, then the natural number $n + 6$ is divisible by six.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.
4. If a brick is on another brick, then the latter brick is on the table.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.

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The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.