Tik-79.144 Logic in computer science: foundations
Examination, December 15, 1999

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \not\models \phi$, then $\Sigma \models \neg\phi$.

(b) Does the following hold: every sentence ϕ which is provable by the tableaux method is valid.

(c) Does the following hold: if A is a set of atomic formulae has two most general unifiers θ and θ', then $A\theta = A\theta'$ holds necessarily.

(d) Does the following hold: propositional logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \leftrightarrow \neg C, A \leftrightarrow B \vee C\} \models B \leftrightarrow A \wedge \neg C$

(b) $\models \exists x(R(x) \wedge \neg R(f(f(x)))) \to \exists x(R(x) \wedge \neg R(f(x)))$

(c) $\models (\forall x(P(x) \to \neg Q(x))) \to ((\exists x\, Q(x)) \to (\forall x\, \neg P(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Natural numbers $0, 1, 2, \ldots$ are represented as ground terms $0, s(0), s(s(0)), \ldots$ built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number n is divisible by two and three, then the natural number $n + 6$ is divisible by six.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.
4. If a brick is on another brick, then the latter brick is on the table.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.