Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

Tik-79.144 Logic in computer science: foundations

Examination, September 20, 1999

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: a proof method M is complete, if every sentence provable by M is valid.

(b) Does the following hold: the composition of substitutions $\{x/f(y), y/z\}$ and $\{x/a, y/b, z/y\}$ is $\{x/f(b), z/y\}$.

(c) Does the following hold: the conjunctive and disjunctive normal forms of a sentence ϕ of predicate logic are logically equivalent.

(d) Does the following hold: if $\{\phi_1, \ldots, \phi_n\} \not\models \phi$, then $\not\models \phi_1 \wedge \ldots \wedge \phi_n \rightarrow \phi$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \rightarrow A, C \rightarrow B, (C \rightarrow A) \rightarrow D\} \models D$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

(c) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x\, Q(x)) \rightarrow (\forall x\, \neg P(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Lists are represented in terms of a binary function symbol c and a constant symbol e such that $c(x, y)$ represents a list that begins with an element x and continues as a list y and e represents the empty list []. In this way, the list $[1, 2]$ gets a representation $c(1, c(2, e))$.

(a) Let the predicate $R(x, y)$ mean that a list x consists of the elements of a list y in the reverse order. Define the predicate R recursively using predicate logic such that your definition covers all lists.

(b) Use resolution to show that the list $[1, 2]$ is the list $[2, 1]$ reversed.

Hint: you'll probably need an additional predicate that allows you to attach an element y at the end of a list x and form a list z in this way.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.

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4. Trees are leaf trees or conifer trees.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.