Tik-79.144 Logic in computer science: foundations

Examination, May 10, 1999

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: the set

$$\{\ P(f(y, g(z)), h(b)),\ \ P(f(h(x), g(a)), x),\ \ P(f(h(b), g(z)), y)\ \}$$

is unifiable (x, y and z are variables).

(b) Does the following hold:
if $\Sigma \cup \{\phi \wedge \neg\psi\}$ is unsatisfiable, then $\Sigma \models \phi \rightarrow \psi$.

(c) Does the following hold: the satisfiability problem of propositional logic belongs to the class **NP**.

(d) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \rightarrow A, C \rightarrow B, (C \rightarrow A) \rightarrow D\} \models D$

(b) $\models \exists x(R(x) \wedge \neg R(f(f(x)))) \rightarrow \exists x(R(x) \wedge \neg R(f(x)))$

(c) $\{\exists x(P(x) \rightarrow Q(x)), \exists x(\neg R(x) \rightarrow \neg Q(x))\} \models \exists x(P(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Represent the following chemical reactions using propositional logic such that you can infer which chemicals can be produced.

$$\begin{array}{l} \mathrm{MgO} + \mathrm{H_2} \rightarrow \mathrm{Mg} + \mathrm{H_2O} \\ \mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2} \\ \mathrm{CO_2} + \mathrm{H_2O} \rightarrow \mathrm{H_2CO_3} \end{array}$$

Transform your sentences to clausal form and use resolution to show that the mutual chemical reactions of magnesium oxide (MgO), hydrogen (H_2), oxygen (O_2) and carbon (C) can produce carbonic acid (H_2CO_3).

Assignment 4 Lists are represented in terms of a binary function symbol c and a constant symbol e such that $c(x, y)$ represents a list that begins with an element x and continues as a list y and e represents the empty list []. In this way, the list $[1, 2]$ gets a representation $c(1, c(2, e))$.

(a) Let the predicate $O(x, y)$ mean that the element x appears in a list y exactly once. Give a recursice definition of the predicate O such that all lists are covered.

(b) Use semantic tableaux to show that the element 1 appears in the list $[2, 1, 3]$ exactly once.

Hint: you need the unique name assumption for the elements of lists.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.