Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

Tik-79.144 Logic in computer science: foundations

Examination, January 8, 1999

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma_1 \models \phi$ and $\Sigma_1 \subseteq \Sigma_2$, then $\Sigma_2 \models \phi$.

(b) Does the following hold: if θ and θ' are two most general unifiers of a set of atomic formulas S, then $\theta = \theta'$.

(c) Does the following hold: the conjunctive and disjunctive normal forms of a sentence ϕ of predicate logic are logically equivalent.

(d) Does the following hold: the satisfiability problem of propositional logic is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \to (B \to C)) \to ((A \to B) \to (A \to C))$

(b) $\{\forall x \exists y R(x,y), R(a,b), \forall x(R(b,x) \to R(x,a))\} \models R(b,a)$

(c) $\{\forall x(\neg P(x) \vee Q(x)),\ \forall x(\neg R(x) \to \neg Q(x))\} \models \forall x(P(x) \to R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Lists are represented in terms of a binary function symbol c and a constant symbol e such that $c(x,y)$ represents a list that begins with an element x and continues as a list y and e represents the empty list []. In this way, the list $[1,2]$ gets a representation $c(1,c(2,e))$.

(a) Let the predicate $R(x,y)$ mean that a list x consists of the elements of a list y in the reverse order. Define the predicate R recursively using predicate logic such that your definition covers all lists.

(b) Use resolution to show that the list $[1,2]$ is the list $[2,1]$ reversed.

Hint: you'll probably need an additional predicate that allows you to attach an element y at the end of a list x and form a list z in this way.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.

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4. Trees are leaf trees or conifer trees.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.