Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

Tik-79.144 Logic in computer science: foundations

Examination, December 7, 1998

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \cup \{\neg\phi\}$ is satisfiable, then $\Sigma \not\models \phi$.

(b) Does the following hold: the empty clause $\square$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(c) Does the following hold: $\{P(f(y), g(z, f(f(y)))), P(x, g(x, z))\}$ is unifiable.

(d) Does the following hold: an existential quantifier can be moved out of an implication as follows: $(\exists x \phi(x) \rightarrow \psi)$ is rewritten as $\forall z(\phi(z) \rightarrow \psi)$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{D \rightarrow \neg B, \neg(\neg A \wedge \neg B), A \rightarrow \neg C, C \wedge D\} \models A \wedge \neg A$

(b) $\models \forall x \exists y R(x, y) \rightarrow (\forall y(\exists x R(y, x) \rightarrow S(y)) \rightarrow \exists x S(x))$

(c) $\{\forall x \forall y \forall z(R(x, y) \wedge R(y, z) \rightarrow R(x, z)), R(a, b)\} \models \neg R(b, a)$.

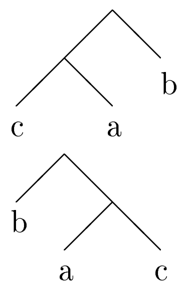
Tableau proofs must contain all intermediary steps !!!

Assignment 3 Let the 3-ary predicate parents(x, y, z) mean that the parents of a person x are y and z. Using this predicate, define the binary predicate relative(x, y) which means that x is a relative of y. Give a resolution proof that Kerttu is a relative of Kustaa using the following database in addition to your definition.

$$\begin{array}{l}\text{parents}(\text{kerttu}, \text{jaakoppi}, \text{hanna})\\ \text{parents}(\text{jaakoppi}, \text{reino}, \text{lahja})\\ \text{parents}(\text{kustaa}, \text{salme}, \text{reino})\end{array}$$

Hint: relatives have an ancestor in common!

Assignment 4 Binary trees are represented in terms of a binary function symbol i (inner nodes) and a unary function symbol l (leaf nodes). In this way, the upper tree in the picture gets a representation $i(i(l(c), l(a)), l(b))$.



(a) Let the predicate $\mathrm{M}(x, y)$ mean that binary tree x is the mirror image of binary tree y. Define the predicate M using sentences of predicate logic such that you can infer whether any given two binary trees are mirror images of each other (assuming the representation given above).

(b) Give a tablaux proof that the upper binary tree is the mirror image of the lower binary tree.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.