Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, August 31, 1998

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\models \phi \vee \psi$, then $\models \phi$ or $\models \psi$.

(b) Does the following hold: the composition of substitutions $\{x/f(y), y/z\}$ and $\{x/a, y/b, z/y\}$ is $\{x/f(b), z/y\}$.

(c) Does the following hold: propositional logic is decidable.

(d) Does the following hold: the sentence $A \wedge B \wedge C$ is the disjunctive normal form of the sentence $\neg(A \rightarrow (\neg B \vee \neg C))$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

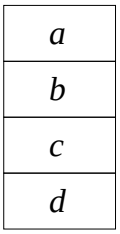
(a) $\{A \wedge \neg B \rightarrow C, C \rightarrow D \wedge \neg D\} \models \neg B \rightarrow \neg A$

(b) $\models \forall x \exists y R(x, y) \rightarrow (\forall y(\neg S(y) \rightarrow \neg \exists x R(x, y)) \rightarrow \exists x S(x))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \neg \exists x(\neg Q(x) \wedge R(x))\} \models \forall x(Q(x) \rightarrow P(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 The stack depicted in the figure is defined using the predicate $O(x, y)$ = "the box x is on the box y" such that only the cases where a box is immediately on another are specified in terms of O.



(a) Define the stack in the figure using the predicate O. Define the predicates $O_n(x, y)$ = "x is the n^{th} box on the box y", when n is $1, 2, \ldots, k$ (for example, $O_3(a, d)$ means that a is the third box on the box d).

(b) Then use resolution to show that the sentence $\exists x \exists y(O_2(a, x) \wedge O_2(y, d) \wedge O_1(y, x))$ is a logical consequence of your definitions.

Assignment 4 Let the 3-ary predicate parents(x, y, z) mean that the parents of a person x are y and z. Using this predicate, define the binary predicate relative(x, y) which means that x is a relative of y. Show that Kerttu is a relative of Kustaa using semantic tableaux and the following database in addition to your definition:

parents(kerttu, jaakoppi, hanna)
parents(jaakoppi, reino, lahja)
parents(kustaa, salme, reino)

Hint: relatives have an ancestor in common.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.