Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, June 8, 1998

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: a sentence ϕ in a Prenex normal form is logically equivalent to its skolemized form ϕ'.

(b) Does the following hold: $\models \phi \wedge \psi$ if and only if $\models \phi$ and $\models \psi$.

(c) Does the following hold: if two substitutions θ and θ' are both most general unifiers of a set of atomic formulas S, then $\theta = \theta'$.

(d) Does the following hold: predicate logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{A \to C, A \vee B, \neg D \to \neg B\} \models \neg C \to D$

(b) $\{\forall x \exists y (P(x) \to Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\models (\forall x (P(x) \to \neg Q(x))) \to ((\exists x\, Q(x)) \to (\exists x\, \neg P(x)))$

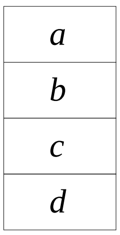
Tableau proofs must contain all intermediary steps !!!

Assignment 3 Derive a conjunctive normal form for the sentence

$$\phi = \forall x \exists y R(x,y) \to (\forall y(\neg S(y) \to \neg \exists x R(x,y)) \to \exists x S(x))$$

and show that ϕ is valid using resolution.

Assignment 4 The stack depicted in the figure is defined using the predicate $O(x,y)$ = "the box x is on the box y" such that only the cases where a box is immediately on another are specified in terms of O.



(a) Define the stack in the figure using the predicate O. Define the predicates $O_n(x,y)$ = "x is the n^{th} box on the box y", when n is $1, 2, \ldots, k$ (for example, $O_3(a,d)$ means that a is the third box on the box d).

(b) Use semantic tableaux to show that the sentence $\exists x \exists y (O_2(a,x) \wedge O_2(y,d) \wedge O_1(y,x))$ is a logical consequence of your definitions.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.