Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, May 20, 1998

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \models \phi$, then the sets of sentences Σ ja $\Sigma \cup \{\phi\}$ have the same models.

(b) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{P(x), Q(x)\}$ ja $\{\neg P(y), \neg Q(y)\}$ by resolution.

(c) Does the following hold: the propositional connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ ja $\leftrightarrow$ are definable in terms of Peirce's arrow $\downarrow$.

(d) Does the following hold: given a structure $\mathcal{A}$ for a language $\mathcal{L}$ of predicate logic, the truth value of every formula $\phi \in \mathcal{L}$ can be determined.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{A \rightarrow D, A \vee B \vee C, \neg D \rightarrow \neg C, \neg B\} \models D$

(b) $\{\forall x(P(x) \rightarrow \exists y Q(x,y)), \exists x P(x), \forall x \forall y(Q(x,y) \rightarrow R(y))\} \models \exists x R(x)$

(c) $\{\forall x \neg P(x), \forall x \exists y(P(x) \vee Q(y))\} \models Q(c)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:

1. Those that are not leaf trees are not alders.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.

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4. Trees are leaf trees or conifer trees.

Use resolution to show that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y) =$ "there is an arc leading from the node x to the node y".

(a) Define the predicates

$$\begin{aligned} C(x,y) &= \text{"there is a connection from the node } x \text{ to the node } y\text{"} \\ \text{and } L(x) &= \text{"the graph has a loop that goes through the node } x\text{"} \end{aligned}$$

by taking the direction of arcs into account.

(b) Describe the directed graph below using the predicate A. Show that $\exists x \exists y(L(x) \wedge A(x,y) \wedge L(y))$ is a logical consequence of your description and the definitions of predicates C and L. Use semantic tableaux.

$$a \ \underset{\longleftarrow}{\overset{\longrightarrow}{}} \ b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.