Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, January 16, 1998

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: a sentence ϕ has at most as many *subsentences* as it has atomic sentences and connectives ($\neg$,$\wedge$,$\vee$,$\rightarrow$,$\leftrightarrow$).

(b) Does the following hold: if $\models \phi \vee \psi$, then $\models \phi$ or $\models \psi$.

(c) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{P(x), \neg Q(x)\}$ and $\{\neg P(y), Q(y)\}$ by resolution.

(d) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \underline{\vee} B) \wedge (B \underline{\vee} C) \rightarrow (A \underline{\vee} C)$ where $\underline{\vee}$ is "exclusive or"

(b) $\{\exists x P(x), \forall x(P(x) \rightarrow Q(x)), \forall x(P(x) \rightarrow R(x))\} \models \exists y(Q(y) \wedge R(y))$

(c) $\{\forall x \forall y(\neg P(x,y) \vee Q(x,y)), \forall x \exists y P(x,y)\} \models \exists y \exists x Q(x,y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x,y)$ = "there is an arc leading from the node x to the node y".

(a) Define predicates $R_n(x,y)$ = "there is a directed route from the node x to the node y such that the route goes through n arcs", when n is $0, 1, 2, \ldots, k$. Describe the graph below using the predicate A.

$$a \quad \begin{array}{c} \longrightarrow \\ \longleftarrow \end{array} \quad b \quad \longrightarrow \quad c$$

(b) Use resolution to show that $\exists x(R_2(x,x) \wedge R_3(x,c))$ is a logical consequence of your description and the definitions of R_2 and R_3.

Assignment 4 Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.

---

4. Trees are leaf trees or conifer trees.

Use semantic tableaux to show that the sentence 4 is a logical consequence of the sentences 1-3.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.