Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, December 18, 1997

Assignment 1 Answer and justify briefly.

(a) Does the following hold: every sentence ϕ of predicate logic can be transformed to a prenex normal form ψ such that $\models \phi \leftrightarrow \psi$.

(b) Does the following hold: $\{P(f(y), g(z, y)), P(x, g(x, z))\}$ is unifiable.

(c) Does the following hold: if $\Sigma \models \phi$ and $\Sigma \subseteq \Gamma$, then $\Gamma \models \phi$.

(d) Does the following hold: if $\mathcal{A} \models \exists x \exists y \neg(x = y)$, then there is at least two elements in the universe A of the structure $\mathcal{A}$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models ((A \to (\neg B \vee \neg C)) \wedge C) \to (B \to \neg A)$

(b) $\{\forall x \exists y R(x, y), \forall x \forall y(\neg S(x) \to \neg R(x, y))\} \models \exists x S(x)$

(c) $\models (\forall x(P(x) \to \neg Q(x))) \to ((\exists x\, Q(x)) \to (\forall x\, \neg P(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 The quantifier $\exists! x$ means that "there is only one x". The claim $\exists! x\, \phi(x)$ can be expressed in predicate logic as the sentence

$$(\exists x \phi(x)) \wedge (\forall x \forall y(\phi(x) \wedge \phi(y) \to x = y)).$$

Formalize the following sentences in predicate logic:

1. There is only one white-bearded.
2. Every Santa Claus is white-bearded.
3. Every white-bearded is Santa Claus.

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4. There is only one Santa Claus.

Give a resolution proof which shows that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y)$ = "there is an arc leading from the node x to the node y".

(a) Define the predicates

$C(x, y)$ = "there is a connection from the node x to the node y"
and $L(x)$ = "the graph has a loop that goes through the node x"

by taking the direction of arcs into account.

(b) Describe the directed graph below using the predicate A. Show that $\exists x(L(x) \wedge C(x, c))$ is a logical consequence of your description and the definitions of predicates C and L. Use semantic tableaux.

$$a \underset{\longleftarrow}{\overset{\longrightarrow}{}} b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.