Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, May 21, 1997

Assignment 1 Answer and justify briefly.

(a) Does the following hold: a proof method M is complete, if every sentence provable by M is valid.

(b) Does the following hold: $\{P(x, f(x, z)), P(h(y), f(z, y))\}$ is unifiable.

(c) Does the following hold: if $\{\phi\} \models \psi$, then $\models \phi \rightarrow \psi$.

(d) Does the following hold: if $\mathcal{A} \models \forall x(x = c \vee x = d) \wedge \neg(c = d)$, there is exactly two elements in the universe A of the structure $\mathcal{A}$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \vee C)) \rightarrow (\neg B \rightarrow (\neg C \rightarrow \neg A))$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

(c) $\{\exists x \exists y P(x, y), \forall x \forall y(P(x, y) \rightarrow Q(x, y))\} \models \exists x Q(x, x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:

1. Alders are leaf trees.
2. Trees are spruces, alders or pines.
3. Spruces and pines are conifer trees.

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4. Trees are leaf trees or conifer trees.

Use resolution to show that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y)$.

(a) There is an n-connection from a node x to a node y, if a route from x to y (in the direction of arcs) goes along n arcs.

Define a predicate $C_n(x, y)$ = "there is an n-connection from node x to node y" where n gets values $0, 1, 2, \ldots, k$.

(b) Describe the directed graph below using the predicate A. Show that $\exists x(C_2(x, x) \wedge C_3(x, c))$ is a logical consequence of your description and the definitions of predicates C_2 and C_3. Use semantic tableaux.

$$a \underset{\longleftarrow}{\longrightarrow} b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.