Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, January 16, 1997

Assignment 1 Answer and justify briefly.

(a) Does the following hold: if $\{\phi_1, \ldots, \phi_n\} \models \phi$, then $\models \phi_1 \wedge \ldots \wedge \phi_n \rightarrow \phi$.

(b) Does the following hold: the sentence $\forall y(P(f(y)) \wedge Q(y))$ is a disjunctive normal form of the sentence $\exists x P(x) \wedge \forall x Q(x)$.

(c) Does the following hold: a sentence ϕ in Prenex normaal form is logically equivalent to its skolemized form ϕ'.

(d) Does the following hold: for all structures $\mathcal{A}$: if $\mathcal{A} \models \forall x(x = c)$, then $\mathcal{A} \models \exists x P(x) \rightarrow \forall y P(y)$.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$

(b) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x\, Q(x)) \rightarrow (\exists x\, \neg P(x)))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \neg\exists x(\neg R(x) \wedge Q(x))\} \models \forall x(P(x) \vee Q(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 A *directed* graph consists of a set $\{a, b, \ldots\}$ of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y)$.

(a) Define a predicate $C(x, y)$ "there is a connection from node x to node y" and a predicate $S(x)$ "the graph has a cycle via x".

(b) Show that there is a cycle via a in the directed graph depicted below. Provide a proof by resolution.

$$a \underset{\longleftarrow}{\longrightarrow} b \longrightarrow c$$

Assignment 4 Formalize the following claims in terms of predicate logic:

1. Birds are hawks, emus or penguins.
2. Penguins and emus do not fly.
3. Hawks and emus do not swim.
4. Tweety is a bird, but not a hawk nor a penguin.

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5. Tweety does not fly nor swim.

Use semantic tableaux to show that the sentence 5 is a logical consequence of the sentences 1-4.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.