Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, December 12, 1996

Assignment 1 Answer and justify briefly.

(a) Does the following hold: for any consistent set of sentences Σ and any sentence ϕ it holds: if $\Sigma \models \phi$, then $\Sigma \not\models \neg\phi$.

(b) Does the following hold: the sentence $\forall x(x = c \vee x = d) \wedge \neg(c = d)$ is true in structures whose universe contains exactly two elements.

(c) Does the following hold: $\{P(x, g(y), f(a)), P(f(y), g(f(z)), z)\}$ is unifiable.

(d) Does the following hold: every sentence ϕ of predicate logic is logically equivalent to some sentence ϕ' in prenex normal form.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C))$

(b) $\{\forall x \exists y(P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\{\forall x(P(x) \rightarrow Q(x) \vee R(x)),\ \neg\exists x R(x)\} \models \forall x(\neg Q(x) \rightarrow \neg P(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Let R be a binary predicate symbol whose interpretation is a relation $R^{\mathcal{A}} \subseteq A \times A$ (the set A is the domain of the structure $\mathcal{A}$). Below there is a table of sentences defining different properties for the relation $R^{\mathcal{A}}$.

Property	Definition
reflexivity	$\forall x R(x, x)$
irreflexivity	$\forall x \neg R(x, x)$
symmetry	$\forall x \forall y(R(x, y) \rightarrow R(y, x))$
asymmetry	$\forall x \forall y(R(x, y) \rightarrow \neg R(y, x))$
connectivity	$\forall x \forall y(R(x, y) \vee R(y, x))$

(a) Let $A = \{1, 2, 3\}$ be the domain. Select $R^{\mathcal{A}}$ such that the sentences defining reflexivity and symmetry are true in $\mathcal{A}$, and the sentence defining connectivity is false in $\mathcal{A}$. Justify your choice.

(b) Prove by resolution that an asymmetric relation is irreflexive.

Assignments continue on the other side !!!

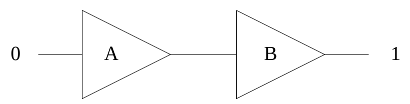
Assignment 4 A buffer is a digital circuit which outputs 1 whenever the input is 1 and which outputs 0 whenever the input is 0.

(a) Define the *normal* operation of a buffer using sentences of predicate logic. Only the the following predicates are allowed:

$$\begin{aligned} B(x) &: \text{“}x \text{ is an buffer”}, \\ I(x,y) &: \text{“the input of } x \text{ is } y\text{” and} \\ O(x,y) &: \text{“the output of } x \text{ is } y\text{”}. \end{aligned}$$

Hint: you should provide (i) sentences that define how the input of a buffer determines the output of the buffer and (ii) sentences that define uniqueness of inputs and outputs (for any digital circuit).

(b) *Extend* the above set of sentences by sentences describing the combination of buffers A and B below with the specified input and output.



Show that the *extended* set of sentences is contradictory using semantic tableaux. What can be concluded upon this inconsistency?

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.