Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, September 2, 1996

Assignment 1 Answer and justify briefly.

(a) Does the following hold: the sentence $A \wedge B \wedge C$ is a disjunctive normal form of the sentence $\neg(A \rightarrow (\neg B \vee \neg C))$.

(b) Does the following hold: if $\{\neg\psi\} \models \neg\phi$ and $\models \phi$, then $\models \psi$ (ϕ and ψ are sentences).

(c) Does the following hold: the sentence $(\exists x P(x)) \leftrightarrow (\forall x P(x))$ is true in structures whose universe contains only one element.

(d) Does the following hold: the sentence ϕ in a Prenex normal form is logically equivalent to its skolemized form ϕ'.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (\neg B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \vee C))$

(b) $\{\exists x P(x), \forall x \exists y (P(x) \rightarrow Q(y))\} \models \exists y Q(y)$

(c) $\{\forall x (P(x) \rightarrow Q(x)),\ \forall x(\neg R(x) \rightarrow \neg Q(x))\} \models \forall x (P(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:

1. Birds are hawks, emus or penguins.
2. Penguins and emus do not fly.
3. Hawks and emus do not swim.
4. Tweety is a bird, but not a hawk nor a penguin.

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5. Tweety does not fly nor swim.

Use resolution to show that the sentence 5 is a logical consequence of the sentences 1-4.

Assignment 4 A *directed* graph consists of a set $\{a, b, \ldots\}$ of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y)$.

(a) Define a predicate $C(x, y)$ "there is a connection from node x to node y" and a predicate $S(x)$ "the graph has a cycle via x".

(b) Show that there is a cycle via a in the directed graph depicted below. Provide a tableaux proof.

$$a \rightleftarrows b \longrightarrow c$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.